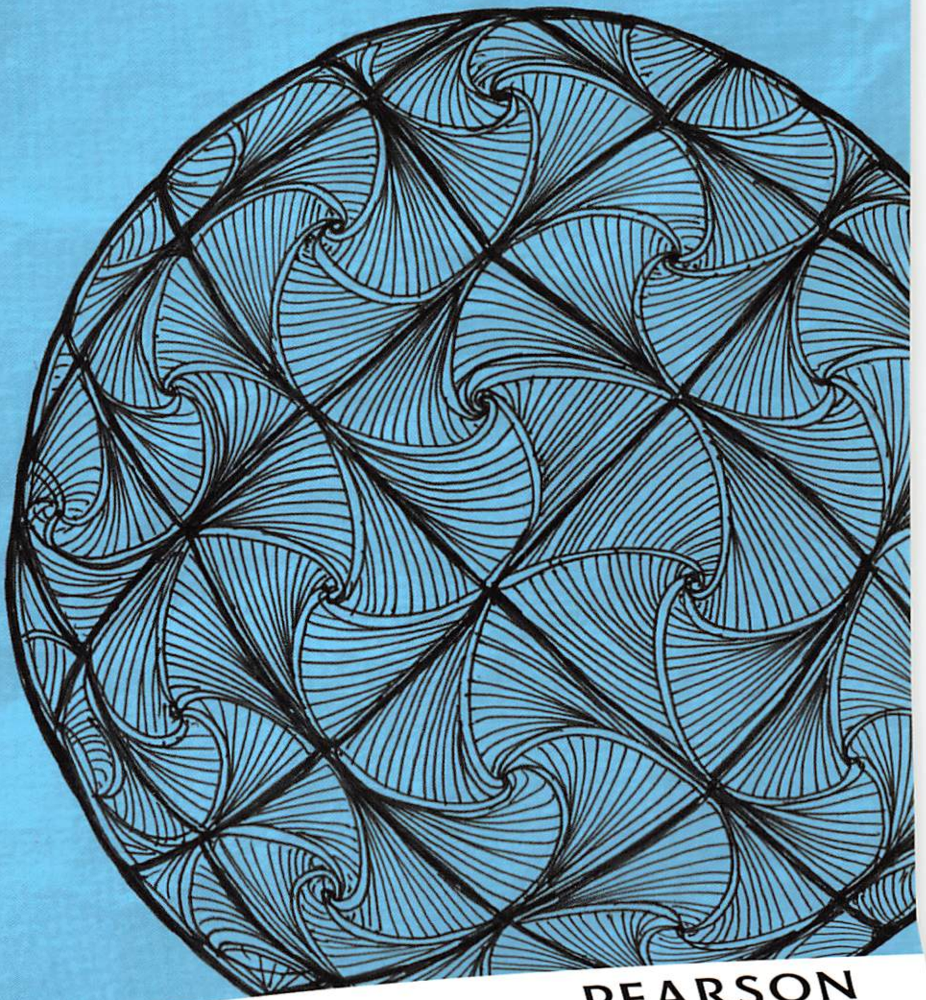
REVISE EDEXCEL GCSE (9–1)

Mathematics

REVISION WORKBOOK

Higher

For the (9–1) exams



PEARSON

Contents

1-to-1 page match with the Revision Guide
ISBN 9781447988090

Number

Algebra

Ratio & Proportion

Geometry & Measures

Probability & Statistics

Practice papers

A small bit of small print
Edexcel publishes Sample Assessment Material and the Specification on its website. This is the official content and this book should be used in conjunction with it. The questions in 'Now try this' have been written to help you practise every topic in the book. Remember: the real exam questions may not look like this.

Had a go ☐ Nearly there ☐ Nailed it! ☐

Algebraic expressions

1 Simplify fully

Guided

(a) $m \times m \times m$

$m^{....}$ **(1 mark)**

(b) $d \times d \times d \times d$

$d^{....}$ **(1 mark)**

(c) $e \times e \times e \times e \times e$

$e^{....}$ **(1 mark)**

2 Simplify

Guided

(a) $x^4 \times x^7$

$x^{.... + ...} = x^{....}$ **(2 marks)**

(b) $y^7 \div y^2$

$y^{.... - ....} = y^{....}$ **(2 marks)**

(c) $t^5 \times t^6 \div t^7$

.... **(2 marks)**

3 Simplify fully

Guided

(a) $(x^3)^2$

$x^{.... \times ....} = x^{....}$ **(1 mark)**

(b) $(y^5)^3$

.... **(2 marks)**

(c) $(t^3)^7$

.... **(2 marks)**

4 Simplify fully

Guided

(a) $\dfrac{x^3 \times x^4}{x^2}$

$\dfrac{x^{.... + ...}}{x^2} = x^{.... - ....}$

$= x^{....}$ **(1 mark)**

(b) $\dfrac{y^{14}}{y^3 \times y^2}$

.... **(2 marks)**

(c) $\left(\dfrac{t^7}{t^4}\right)^2$

.... **(2 marks)**

5 Simplify fully

(a) $7xy^3 \times 4x^2y^4$

.... **(2 marks)**

(b) $\dfrac{16x^4y^3}{8xy^2}$

.... **(2 marks)**

(c) $(3x^2y^5z^3)^4$

.... **(2 marks)**

6 Simplify fully

(a) $(25x^6)^{\frac{1}{2}}$

.... **(2 marks)**

(b) $(16x^3y^4)^{\frac{3}{2}}$

.... **(2 marks)**

(c) $(81x^5y^3)^{\frac{1}{4}}$

.... **(2 marks)**

7 Simplify fully

(a) $\left(\dfrac{1}{3x^4}\right)^{-2}$

.... **(2 marks)**

(b) $\left(\dfrac{25}{64x^4y^{10}}\right)^{-\frac{1}{2}}$

.... **(2 marks)**

(c) $\left(\dfrac{27}{64x^3y^9}\right)^{-\frac{2}{3}}$

.... **(2 marks)**

Expanding brackets

1 Expand and simplify

Guided

(a) $(x + 3)(x + 4)$

$x(x + 4) + 3(x + 4)$

$= x^{....} + ...x + ...x + 12$

$= x^{....} + ...x + 12$

(2 marks)

(b) $(x + 5)(x - 3)$

$x(x - 3) + 5(x - 3)$

$= x^{....} - ....x + ...x - ....$

$= x^{....} + ...x - ....$

(2 marks)

(c) $(x - 2)(x - 6)$

(2 marks)

2 Expand and simplify

Guided

(a) $(x + 3)^2$

$(x + 3)(x + 3)$

$= x(x + 3) + 3(x + 3)$

$= x^{....} + ...x + ...x + ...$

$= x^{....} + ...x + ...$

(2 marks)

(b) $(x - 4)^2$

(2 marks)

(c) $(2x + 1)^2$

(2 marks)

3 Expand and simplify

Multiply out the brackets first and then multiply this expression by x.

Guided

(a) $x(x + 3)(x + 5)$

$x(x + 3)(x + 5)$

$= x(x^{....} + ...x + ...x + ...)$

$= x(x^{....} + ...x + ...)$

$= x^{....} + ...x^{....} + ...x$

(2 marks)

(b) $x(x - 2)(x + 4)$

.... **(2 marks)**

(c) $x(x - 3)(x - 7)$

.... **(2 marks)**

4 Expand and simplify

(a) $(x + 3)^3$

.... **(2 marks)**

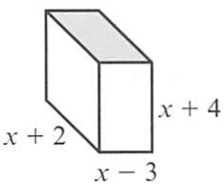
(b) $(x - 4)^3$

.... **(2 marks)**

(c) $(2x + 1)^3$

.... **(2 marks)**

5 The diagram shows a cuboid with a length of $x + 2$, a width of $x - 3$, and a height of $x + 4$. All measurements are in cm.



(a) Write down an expression, in terms of x, for the total surface area.

.... **(2 marks)**

(b) Show that the volume of the cuboid can be written in the form $ax^3 + bx^2 + cx + d$ cm^3.

(2 marks)

Had a go ☐ Nearly there ☐ Nailed it! ☐

Factorising

1 Factorise

(a) $3x + 6$

Guided

$= 3(\ldots\ldots + \ldots\ldots)$ **(1 mark)**

(b) $2p - 6$

.... **(1 mark)**

(c) $5y - 15$

$= 5(\ldots\ldots - \ldots\ldots)$ **(1 mark)**

2 Factorise

(a) $x^2 + 6x$

Guided

$= x(\ldots\ldots + \ldots\ldots)$ **(1 mark)**

(b) $x^2 + 4x$

.... **(1 mark)**

(c) $x^2 - 12x$

$= x(\ldots\ldots - \ldots\ldots)$ **(2 marks)**

3 Factorise fully

(a) $3p^2 + 6p$

Guided

$= 3p(\ldots\ldots + \ldots\ldots)$ **(1 mark)**

(b) $8y^2 - 24y$

.... **(2 marks)**

'Factorise fully' means that you need to take out the highest common factor (HCF).

4 Factorise fully

(a) $4d^2 + 12d$

.... **(2 marks)**

(b) $6x^2 - 18x$

.... **(2 marks)**

If you wrote $4d^2 + 12d = 4(d^2 - 3d)$ you would not have factorised fully, because 4 is not the HCF of both terms.

5 Factorise

Guided

(a) $x^2 + 4x + 3$

$\ldots \times \ldots = +3$

$\ldots + \ldots = +4$

$x^2 + 4x + 3 = (x + \ldots)(x + \ldots)$

(2 marks)

(b) $x^2 + 11x + 10$

$\ldots \times \ldots = +10$

$\ldots + \ldots = +11$

$x^2 + 11x + 10 = (x \ldots\ldots\ldots)(x \ldots\ldots\ldots)$

(2 marks)

You need to find two numbers that multiply to give 3 and add up to give 4.

6 Factorise

(a) $x^2 + 6x - 7$

.... **(2 marks)**

(b) $x^2 + 4x - 5$

.... **(2 marks)**

(c) $x^2 - 2x - 15$

.... **(2 marks)**

7 Factorise

Guided

(a) $x^2 - 9$

$a = x,\ b = 3$

$x^2 - 9 = (x + \ldots)(x - \ldots)$

(2 marks)

(b) $x^2 - 144$

.... **(2 marks)**

This is a difference of two squares. You can use the rule $a^2 - b^2 = (a + b)(a - b)$.

8 Factorise

(a) $3x^2 - 7x + 2$

.... **(2 marks)**

(b) $2x^2 - x - 3$

.... **(2 marks)**

(c) $3x^2 - 16x - 12$

.... **(2 marks)**

Linear equations 1

1 Solve

(a) $3x + 1 = 13$ (b) $5x - 3 = 27$ (c) $26 = 7q - 9$

Guided

$3x + 1 = 13$ (-1)

$3x = 13 - 1$

$3x = ...$ $(\div 3)$

$x = ...$ **(1 mark)** $x = ...$ **(1 mark)** $q = ...$ **(1 mark)**

(d) $12x + 18 = 66$ (e) $\frac{t}{6} - 7 = 3$ (f) $\frac{d}{3} + 2 = -4$

$x = ...$ **(1 mark)** $t = ...$ **(1 mark)** $d = ...$ **(1 mark)**

2 Solve

Multiply out the brackets.

(a) $3(3x + 5) = 42$ (b) $5(2x + 3) = 35$ (c) $5(x - 3) = -25$

Guided

$9x + ... = 42$

$9x = 42 - ....$

$9x = ...$ $(\div 9)$

$x = ...$ **(2 marks)** $x = ...$ **(2 marks)** $x = ...$ **(2 marks)**

(d) $4(5x + 7) = 16$ (e) $3(4x + 13) = 51$ (f) $3(10 - 4x) = 45$

$x = ...$ **(2 marks)** $x = ...$ **(2 marks)** $x = ...$ **(2 marks)**

3 Solve

Collect all the x terms on one side.

(a) $2x + 3 = x + 7$ (b) $7y + 15 = 4y - 6$ (c) $4t - 6 = 2t + 18$

$x = ...$ **(2 marks)** $y = ...$ **(2 marks)** $t = ...$ **(2 marks)**

(d) $2(x + 3) = x + 10$ (e) $5(x - 4) = 3(x + 2)$ (f) $3(2y - 4) = 2(6 - 3y)$

$x = ...$ **(2 marks)** $x = ...$ **(2 marks)** $y = ...$ **(2 marks)**

4 Carl buys 8 bags of marbles. Each bag contains m marbles. He plays his friend and wins another 7 marbles. When Carl gets home, he counts his marbles and finds that he has 103 marbles altogether. Calculate the value of m. You must show all of your working.

You can write an equation involving m. Carl starts with $8m$ marbles. He then adds 7 to get 103.

$m = ...$ **(3 marks)**

Had a go ☐ Nearly there ☐ Nailed it! ☐

Linear equations 2

Guided

1 Solve

(a) $\frac{3x+10}{2} = 12$ (b) $\frac{2x+7}{5} = 3$ (c) $4 = \frac{5x-3}{3}$

$\frac{3x + 10}{2} = 12 \quad (\times 2)$

$\frac{2(3x + 10)}{2} = 12 \times 2$

$3x + 10 = \ldots\ldots \quad (-10)$

$3x = \ldots\ldots - \ldots\ldots$

$3x = \ldots\ldots \quad (\div 3)$

$x = \ldots\ldots\ldots$ **(3 marks)** $x = \ldots\ldots\ldots$ **(3 marks)** $x = \ldots\ldots\ldots$ **(3 marks)**

Guided

2 Solve

(a) $\frac{6-2x}{4} = 2 - x$ (b) $\frac{5x}{3} - 6 = x + 2$

Place brackets around 2 – *x*.

Multiply out the brackets.

Collect all the *x* terms on one side.

$\frac{6 - 2x}{4} = 2 - x \quad (\times 4)$

$\frac{4(6 - 2x)}{4} = 4(2 - x)$

$6 - 2x = 4(2 - x)$

$6 - 2x = \ldots\ldots - \ldots\ldots x$

$\ldots\ldots x - 2x = \ldots\ldots - 6$

$\ldots\ldots x = \ldots\ldots\ldots\ldots\ldots\ldots$ **(3 marks)** $x = \ldots\ldots\ldots\ldots\ldots\ldots$ **(3 marks)**

Guided

3 Solve

(a) $\frac{x-5}{5} - \frac{4-x}{3} = 5$ (b) $\frac{3x-1}{2} - \frac{2(4+3x)}{13} = 2$

$\frac{3(x - 5)}{5 \times 3} - \frac{5(4 - x)}{3 \times 5} = 5$

$\frac{3x - \ldots\ldots - \ldots\ldots + \ldots\ldots x}{15} = 5 \quad (\times 15)$

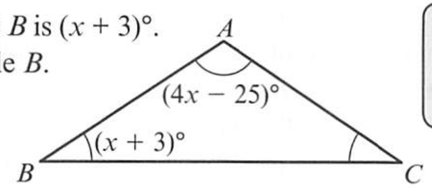
$\ldots\ldots x - \ldots\ldots = 5 \times 15$

$\ldots\ldots x - \ldots\ldots = 75$

$\ldots\ldots x = \ldots\ldots$

$\ldots\ldots x = \ldots\ldots\ldots\ldots\ldots\ldots$ **(3 marks)** $x = \ldots\ldots\ldots\ldots\ldots\ldots$ **(3 marks)**

4 *ABC* is a triangle. Angle *A* is $(4x - 25)°$ and angle *B* is $(x + 3)°$.
The size of angle *A* is three times the size of angle *B*.
Work out the value of *x*.



Remember to set up an equation.

$x = \ldots\ldots\ldots\ldots\ldots\ldots$ **(3 marks)**

Formulae

Guided

1 Using the formula $y = 3x - 7$, find the value of y when

(a) $x = 5$

Substitute $x = 5$ into the formula.

$y = 3(5) - 7$

$= ..... - 7 =$ .... **(2 marks)**

(b) $x = -4$

$y = 3(.....) - 7$

$= ..... - 7 =$ .... **(2 marks)**

Guided

2 The value of y can be found by using the formula $y = 2x^2 + 5$. Work out the value of y when

(a) $x = 3$

$y = 2(.....)^2 + 5$

$= ..... + 5 =$ .... **(2 marks)**

(b) $x = \frac{5}{2}$

$y =$ .... **(3 marks)**

Guided

3 Give answers to parts (a) to (c) correct to 3 significant figures.

(a) Using the formula $V = \frac{1}{3}\pi r^2 h$, work out the value of V when $r = 24$ and $h = 5.8$.

$V = \frac{1}{3}\pi(.....)^2 \times ..... =$ .... **(2 marks)**

(b) Using the formula $s = ut + \frac{1}{2}at^2$, work out the value of s when $u = -4$, $t = 6$ and $a = -9.8$.

$s =$ .... **(3 marks)**

(c) Using the formula $T = 2\pi\sqrt{\frac{L}{g}}$, work out the value of T when $L = 19.4$ and $g = 9.8$.

$T =$ .... **(2 marks)**

(d) Using the formula $T = \frac{2Mmg}{M + m}$, work out the value of T when $M = 3.6$, $m = 1.4$ and $g = 9.8$.
Give your answer correct to 2 significant figures.

$T =$ .... **(2 marks)**

4 The number, N, of plants per hectare is given by the formula $N = \frac{12\,000}{dx}$, where d is the distance between the rows of plants in metres and x is the spacing between the plants in metres.
Given that $d = 0.85$ and $x = 0.54$, work out the value of N.
Give your answer correct to 2 significant figures.

$N =$ .... **(3 marks)**

PROBLEM SOLVED!

5 The surface area of a cylinder is given by the formula
$A = 2\pi r^2 + 2\pi rh$
The volume of a cylinder is given by the formula $V = \pi r^2 h$
Find a formula for A in terms of r and V.

You will need to use problem-solving skills throughout your exam – **be prepared!**

$A =$ .... **(3 marks)**

Arithmetic sequences

1 Here are some sequences. Find an expression for the *n*th term of each linear sequence.

Guided

(a) 5 9 13 17

+4 +4 +4

> Work backwards to find the zero term of the sequence. You need to subtract the difference from the first term. Then *n*th term = difference × *n* + zero term.

*n*th term = 4*n* .... **(2 marks)**

(b) 2 5 8 11

.... **(2 marks)**

(c) 2 9 16 23

.... **(2 marks)**

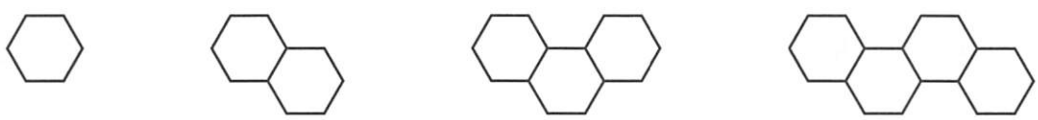
2 Here are the first five terms of a linear sequence.

4 7 10 13 16

Find an expression, in terms of *n*, for the *n*th term of the linear sequence.

.... **(2 marks)**

3 Here are some patterns made from sticks.



pattern number 1 pattern number 2 pattern number 3 pattern number 4

Write down a formula for the number of sticks, *S*, in terms of the pattern number, *n*.

S = ... **(3 marks)**

4 Here are the first four terms of an arithmetic sequence.

3 7 11 15 19

Guided

(a) Find an expression, in terms of *n*, for the *n*th term of the sequence.

.... *n* − .... **(2 marks)**

(b) Molly says that 199 is a term in the arithmetic sequence. Is Molly correct?
Give a reason for your answer.

> Set the *n*th term of the sequence equal to 199 and solve the equation to find *n*.
> If *n* is an integer then the term is part of the sequence.
> If *n* is not an integer then the term is NOT part of the sequence.

.... **(2 marks)**

5 Here are the first five terms of a linear sequence.

3 9 15 21 27

(a) Write down an expression, in terms of *n*, for the *n*th term of this sequence.

.... **(2 marks)**

(b) Conor says that 242 is a term in the linear sequence. Is Conor correct?
Give a reason for your answer.

.... **(2 marks)**

Solving sequence problems

1 The nth term of an arithmetic sequence is given by $an + b$, where a and b are integers. The 4th term is 11 and the 7th term is 20. Work out the values of a and b.

Guided

Set up two equations and solve them simultaneously.

$11 = a \times$ ………… $+ b$ and $20 = a \times$ ………… $+ b$

……………………………………………… **(2 marks)**

2 The rule for finding the next term in a sequence is | Multiply by k then add 3 |

The second term of the sequence is 21 and the third term is 45.

(a) Work out the first term of the sequence.

……………………………………………… **(3 marks)**

(b) Write down an expression for u_{n+1} in terms of u_n.

……………………………………………… **(1 mark)**

3 The rule for finding the next term in a sequence is | Add 6 then divide by p |
The second term is 15 and the third term is 7.
Work out the first term of the sequence.

……………………………………………… **(4 marks)**

4 Here are the first seven terms of a Fibonacci sequence.

1 1 2 3 5 8 13

To find the next term in the sequence, add the two previous terms.

(a) Work out the tenth term of the Fibonacci sequence.

……………………………………………… **(1 mark)**

The first three terms of a different Fibonacci sequence are

$zx \quad y \quad x + y$

(b) Work out the fifth term of the sequence.

……………………………………………… **(2 marks)**

The third term of the sequence is 11 and the fifth term of the sequence is 28.

(c) Work out the value of x and the value of y.

$x =$ ……………………… and $y =$ ……………………… **(3 marks)**

5 3 $3\sqrt{3}$ 9 $9\sqrt{3}$ 27

Write down the next two terms of the sequence.

……………………………………………… **(2 marks)**

Quadratic sequences

Guided

1 A quadratic sequence is given by $u_n = n^2 + 2n - 1$

Write down the first six terms of the sequence.

Substitute n = 1, 2, 3, 4, 5 and 6 into the formula.

$u_1 = 1^2 + 2(1) - 1 =$ .... $u_2 = (....)^2 + 2(....) - 1 =$ ....

$u_3 = (....)^2 + 2(....) - 1 =$ .... $u_4 = (....)^2 + 2(....) - 1 =$ ....

$u_5 = (....)^2 + 2(....) - 1 =$ .... $u_6 = (....)^2 + 2(....) - 1 =$ .... **(2 marks)**

2 Write down the formula for the nth term for each of these quadratic sequences.

(a) 3 6 11 18 27 38

.... **(3 marks)**

(b) 3 13 27 45 67 93

.... **(3 marks)**

(c) 4 10 20 34 52 74

.... **(3 marks)**

(d) 2 9 22 41 66 97

.... **(3 marks)**

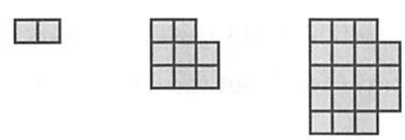
(e) 2 12 26 44 66 92

.... **(3 marks)**

(f) 1 5 15 31 53 81

.... **(3 marks)**

3 Here are some patterns made from square slabs.



(a) Write down an expression, in terms of n, for the nth term of this sequence.

.... **(3 marks)**

(b) Jane says that 75 is a term in the quadratic sequence. Is Jane correct? Give a reason for your answer.

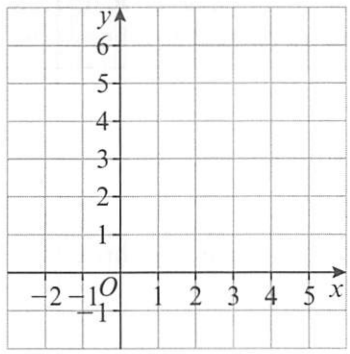
.... **(1 mark)**

Straight-line graphs 1

Guided

1 On the grid draw the graph of $x + y = 5$ for the values of x from −2 to 5.

First draw a table of values. The question tells you to use 'values of x from −2 to 5'. Next work out the values of y.



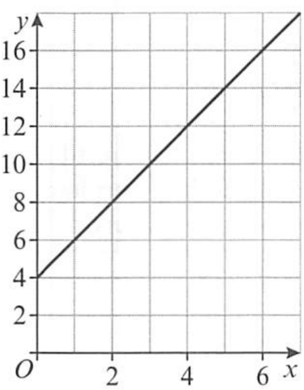
x	−2	−1	0	1	2	3	4	5
y		6						

(2 marks)

Guided

2 Find the equation of the straight line below.

Draw a triangle on the graph and use it to find the gradient.



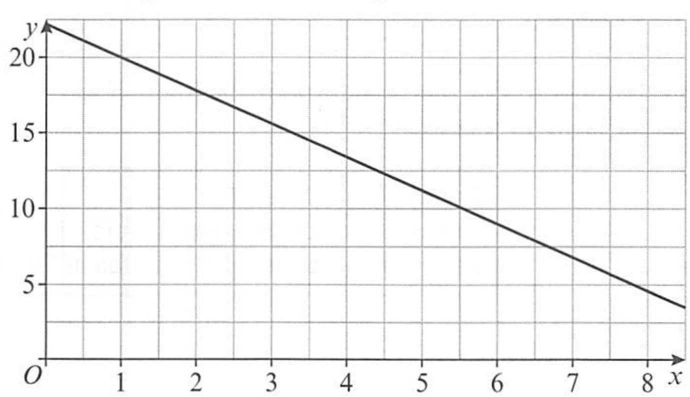
$$\text{Gradient} = \frac{\text{distance up}}{\text{distance across}} = \text{........................}$$

Use $y = mx + c$ to find the equation of the line, where m is the gradient and c is the y-intercept.

.... **(2 marks)**

3 Find the equation of the straight line.

The graph slopes down so the gradient will be negative.



.... **(2 marks)**

Had a go ☐ Nearly there ☐ Nailed it! ☐

Straight-line graphs 2

Guided

1 Find the equation of the straight line with

Substitute the value of the gradient into $y = mx + c$. Then substitute the x- and y-values given into your equation. Solve the equation to find c. Remember to write your completed equation at the end.

(a) gradient 3, passing through the point (2, 5)

$y = 3x + c$

$5 = \ldots\ldots \times \ldots\ldots + c$

$c = \ldots\ldots$

$y = \ldots\ldots x \ldots\ldots\ldots\ldots\ldots\ldots$ **(2 marks)**

(b) gradient −2, passing through the point (3, 6)

.... **(2 marks)**

(c) gradient 4, passing through the point (−2, 7)

.... **(2 marks)**

Guided

2 Find the equation of the straight line which passes through the points

(a) (3, 2) and (5, 6)

Work out the gradient and then substitute x- and y-values for one of the points into the equation $y = mx + c$ to work out c.

$m = \dfrac{\ldots\ldots - \ldots\ldots}{\ldots\ldots - \ldots\ldots} = \ldots\ldots\ldots\ldots\ldots\ldots$

$2 = \ldots\ldots \times 3 + c$

$c = \ldots\ldots$

$y = \ldots\ldots\ldots\ldots\ldots\ldots$ **(3 marks)**

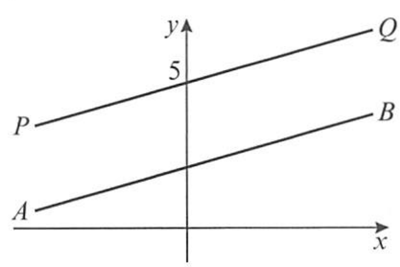
(b) (2, 5) and (4, 9)

.... **(3 marks)**

(c) (−1, −2) and (−4, −8)

.... **(3 marks)**

3 Here are two straight lines. AB is parallel to PQ. The equation of the line AB is $y = 4x + 1$. Find the equation of the straight line PQ.



Find the gradient of AB.
Find the value of the y-intercept for PQ.
Use $y = mx + c$ to find the equation of PQ.

.... **(2 marks)**

4 A straight line passes through the points with coordinates (−1, −1), (3, 15) and (k, 23).
Work out the value of k. You must show all your working.

First work out the gradient of the line using the given points.

$k = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ **(4 marks)**

Parallel and perpendicular

1 Here are the equations of five straight lines.

P: $y = 2x + 7$ **Q**: $y = -2x + 7$ **R**: $y = x + 7$ **S**: $y = -\frac{1}{2}x + 8$ **T**: $y = \frac{1}{2}x + 4$

(a) Write down the letter of the line that is parallel to $y = x + 4$

When two lines are parallel their gradients are the same.

.... **(1 mark)**

(b) Write down the letter of the line that is perpendicular to $y = 2x - 3$

When the gradient of a line is m, the gradient of a perpendicular line is $-\frac{1}{m}$.

.... **(1 mark)**

2 (a) A straight line **L** is parallel to $y = 3x - 4$ and passes through the point (4, 5). Find the equation of line **L**.

Guided

m = ...

...... = ...(......) + c

Compare the straight line with $y = mx + c$ to find the value of m.

Rearranging for c

c = ...

Hence, y = ...x .... **(3 marks)**

(b) Put a tick (✓) beside the equation which is the equation of a straight line that is perpendicular to the line with equation $y = 3x - 4$.

$y = 3x - 4$	$y = 4 - 3x$	$y = \frac{1}{3}x - 4$	$y = 4 - \frac{1}{3}x$	$y = 3x + 4$

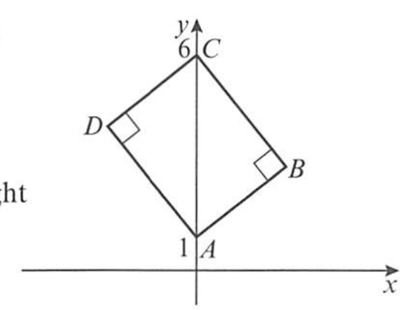
(1 mark)

3 A straight line **L** passes through the point with coordinates (3, 7) and is perpendicular to the line with equation $y = 3x + 5$. Find the equation of the line **L**.

.... **(3 marks)**

4 $ABCD$ is a rectangle. A is the point (0, 1) and C is the point (0, 6). The equation of the straight line through A and B is $y = 2x + 1$

(a) Find the equation of the straight line through D and C.



.... **(2 marks)**

(b) Find the equation of the straight line through B and C.

.... **(2 marks)**

5 The point P has coordinates (2, 1) and the point Q has coordinates (−2, −1). Find the equation of the perpendicular bisector of PQ.

.... **(4 marks)**

Had a go ☐ Nearly there ☐ Nailed it! ☐

Quadratic graphs

Guided

1 (a) Complete the table of values for $y = x^2 - 2$

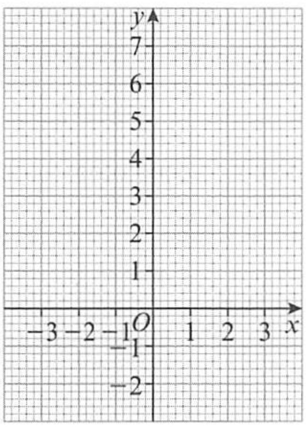
x	−3	−2	−1	0	1	2	3
y		2					7

Substitute each value of x into the equation $y = x^2 - 2$ to find the value of y.

$x = -3$: $y = (-3 \times -3) - 2 =$ ...... $- 2 =$ ....

$x = -1$: $y = (-1 \times$ ......$) - 2 =$ ...... $- 2 =$ .... **(2 marks)**

(b) On the grid draw the graph of $y = x^2 - 2$



(1 mark)

(c) Write down the coordinates of the turning point.

The turning point is the point where the direction of the curve changes.

.... **(2 marks)**

Guided

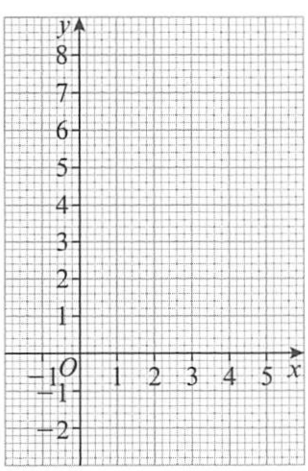
2 (a) Complete the table of values for $y = x^2 - 4x + 3$

x	−1	0	1	2	3	4	5
y		3					8

$x = -1$: $y = (-1 \times -1) - (4 \times -1) + 3 =$ ....

$x = 1$: $y = (1 \times$ ......$) - (4 \times$ ......$) + 3 =$ .... **(2 marks)**

(b) On the grid draw the graph of $y = x^2 - 4x + 3$



(2 marks)

(c) Write down the coordinates of the turning point.

.... **(1 mark)**

Cubic and reciprocal graphs

Guided

1 (a) Complete the table of values for $y = x^3 - 4x - 2$

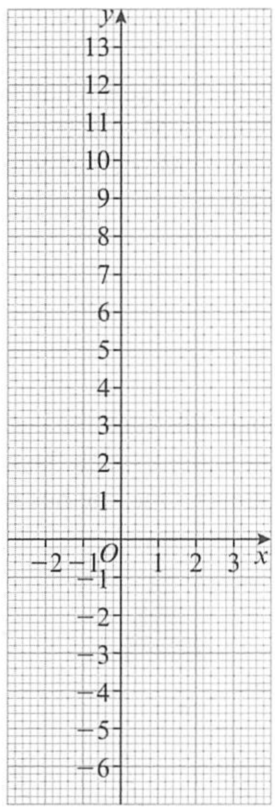
x	−2	−1	0	1	2	3
y			−2			13

Substitute each value for x into the rule $y = x^3 - 4x - 2$ to find the value of y.

$x = -2$: $y = (-2 \times -2 \times -2) - (4 \times -2) - 2 =$ ....

$x = -1$: $y = (-1 \times -1 \times -1) - (4 \times -1) - 2 =$ .... **(2 marks)**

(b) On the grid draw the graph of $y = x^3 - 4x - 2$



(2 marks)

(c) Write down the coordinates of the turning points.

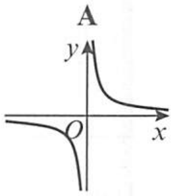
.... **(2 marks)**

(d) Use your graph to find estimates of the solutions to the equation $x^3 - 4x - 2 = -3$

Draw the line $y = -3$ on the graph. Find the x-coordinates of the points of intersection with the curve.

.... **(2 marks)**

2

A


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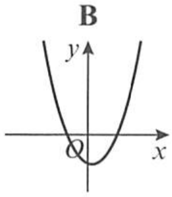
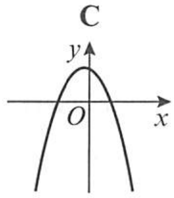
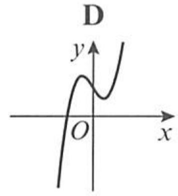
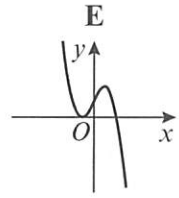


C


D


E


Write down the letter of the graph which could have the equation

(i) $y = x^2 - x - 6$

.... **(1 mark)**

(ii) $y = x^3 - 3x + 5$

.... **(1 mark)**

(iii) $y = \frac{1}{x}$

.... **(1 mark)**

(iv) $y = 6 - x - x^2$

.... **(1 mark)**

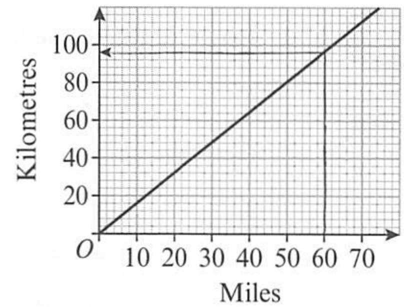
(v) $y = 2 + 3x - x^3$

.... **(1 mark)**

Real-life graphs

Guided

1 You can use this graph to convert between miles and kilometres.



(a) Use the graph to change 60 miles into kilometres.

60 miles = ... kilometres

Draw a vertical line from 60 miles up to the line and then draw a horizontal line across to the kilometres axis.

(1 mark)

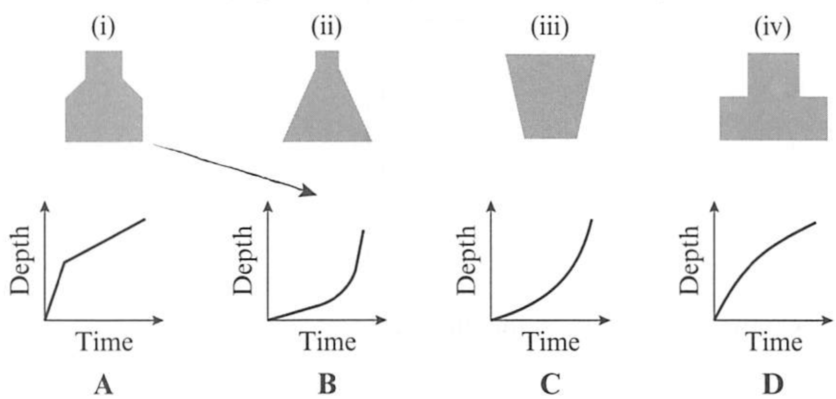
(b) The distance from Rome to Lyon is 660 miles. The distance from Rome to Marseille is 950 km. Is Rome closer to Lyon or Marseille? You must show all of your working.

Convert 60 miles to km. Using this information work out 600 miles and then add on the conversion for 60 miles.

(3 marks)

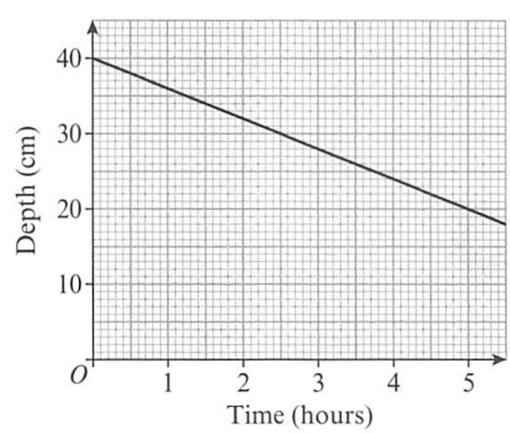
Guided

3 Here are four flasks. Rachael fills each flask with water. The graphs show the rate of change of the depth of the water in each flask as Rachael fills it. Draw a line from each flask to the correct graph. One line has been drawn for you.



(2 marks)

3 Water flows out of a cylindrical tank at a constant rate. The graph shows how the depth of water in the tank varies with time.



(a) Work out the gradient of the straight line.

.... **(2 marks)**

(b) Give a practical interpretation of the value you worked out in part (a).

.... **(1 mark)**

Quadratic equations

Guided

1 Solve

(a) $x^2 - 3x = 0$

$x(x - \ldots\ldots) = 0$

$x = 0$ or $x = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$ **(2 marks)**

(b) $2x^2 + 5x = 0$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ **(2 marks)**

> Find the values of x that make each factor equal to 0. The first factor is just x so one solution is $x = 0$.

Guided

2 Solve

(a) $x^2 - 4 = 0$

$(x + \ldots\ldots)(x - \ldots\ldots) = 0$

$x = \ldots\ldots$ or $x = \ldots\ldots$ **(2 marks)**

(b) $16x^2 - 49 = 0$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ **(2 marks)**

(c) $2x^2 - 50 = 0$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ **(2 marks)**

Guided

3 Solve

(a) $x^2 + 6x + 8 = 0$

$(x + 2)(x + \ldots\ldots) = 0$

$x = -2$ or $x = \ldots\ldots$ **(2 marks)**

(b) $x^2 - 7x + 12 = 0$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ **(2 marks)**

(c) $x^2 = 10x - 21$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ **(2 marks)**

4 Solve

(a) $2x^2 + 7x + 3 = 0$

$(2x + 1)(x + \ldots\ldots) = 0$

$x = \ldots\ldots$ or $x = \ldots\ldots$ **(2 marks)**

> The first factor is $2x + 1$, so the first solution is $x = -\frac{1}{2}$.

(b) $3x^2 + 5x + 2 = 0$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ **(2 marks)**

5 A rectangle has length $(x + 5)$ cm and width $(x - 2)$ cm. The area of the rectangle is 60 cm^2.

(a) Show that $x^2 + 3x - 70 = 0$.

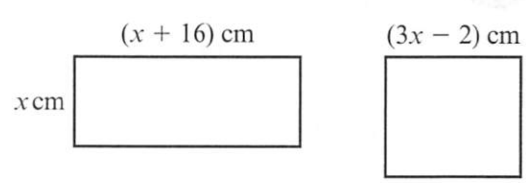
> Use information given to form an equation for the area. Then rearrange the equation to the form given.

(2 marks)

(b) Solve the equation to find the length and width of the rectangle.

(3 marks)

6 The rectangle shown has length $(x + 16)$ cm and width x cm. The square has length $(3x - 2)$ cm. The area of the rectangle is 16 cm^2 more than the area of the square. The area of the square is more than 1 cm^2. Find the value of x.



$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ **(4 marks)**

Had a go ☐ Nearly there ☐ Nailed it! ☐

The quadratic formula

1 How many solutions do the following quadratics have?

Use $b^2 - 4ac$ to determine the number of solutions.

(a) $x^2 - 2x + 1 = 0$ (b) $11x^2 + 2x - 7 = 0$ (c) $3x^2 + 5x + 9 = 0$

.... **(2 marks)** .... **(2 marks)** .... **(1 mark)**

2 Solve these quadratic equations and give your answers correct to 3 significant figures.

(a) $x^2 - 5x + 2 = 0$

Guided

Use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1$, $b = -5$, $c = 2$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - (4 \times 1 \times 2)}}{2 \times 1}$$

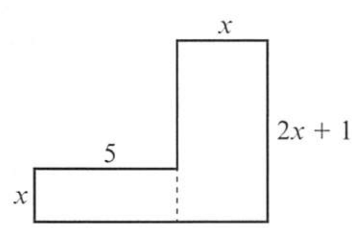
$= \dfrac{\text{..........} + \sqrt{\text{..........}}}{\text{..........}}$ or $\dfrac{\text{..........} - \sqrt{\text{..........}}}{\text{..........}}$ = ... or ....

= .... or .... (to 3 s.f.) **(3 marks)**

(b) $2x^2 + 3x - 1 = 0$ (c) $5x^2 - 4x - 2 = 0$ (d) $2 - 3x - x^2 = 0$

.... **(3 marks)** .... **(3 marks)** .... **(3 marks)**

3 The diagram shows a six-sided shape.
All the corners are right angles.
All the measurements are given in cm.
The area of the shape is 95 cm^2.



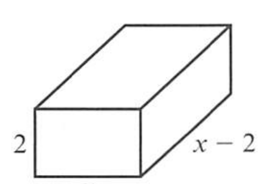
(a) Show that $2x^2 + 6x - 95 = 0$.

(3 marks)

(b) Work out the value of x. Give your answer correct to 3 significant figures.

x = ... **(4 marks)**

4 The diagram shows a cuboid.
All the measurements are in cm.
The volume of the cuboid is 51 cm^3.



(a) Show that $2x^2 - 4x - 51 = 0$. (b) Find the value of x.

(3 marks) x = ... **(3 marks)**

(c) What assumption have you made about the value of x?

.... **(1 mark)**

Completing the square

1 Write the following in the form $(x + p)^2 + q$

(a) $x^2 - 10x + 1$ (b) $x^2 - 2x + 7$ (c) $x^2 + 6x - 4$

Guided

$x^2 - 10x + 1$

$= (x - \text{..........})^2 - \text{..........} + 1$

$= (x - \text{..........})^2 - \text{..........}$ ....

(2 marks) **(2 marks)** **(2 marks)**

2 Write the following in the form $a(x + p)^2 + q$

(a) $2x^2 + 12x + 9$ (b) $3x^2 + 6x - 2$ (c) $4x^2 + 8x - 11$

Guided

$2x^2 + 12x + 9$

$= 2(x^2 + 6x) + 9$

$= 2(x + \text{..........})^2 - \text{..........} + 9$

$= 2(x + \text{..........})^2 - \text{..........}$ ....

(2 marks) **(2 marks)** **(2 marks)**

3 Solve, giving your answers to 3 significant figures

> Rewrite the equation in completed square form, and then solve. Remember that any positive number has two square roots: one positive and one negative.

(a) $x^2 + 4x - 6 = 0$ (b) $x^2 - 7x + 3 = 0$ (c) $4x^2 + 7x - 2 = 0$

.... **(2 marks)** .... **(2 marks)** .... **(2 marks)**

4 Solve, giving your answers in surd form

(a) $x^2 + 4x - 3 = 0$ (b) $2x^2 + 5x - 6 = 0$

.... **(2 marks)** .... **(2 marks)**

5 (a) Find the values of p and q such that $3x^2 + 5x + 1 = a(x + p)^2 + q$

$p =$ ....

$q =$ .... **(2 marks)**

(b) Hence, or otherwise, solve the equation $3x^2 + 5x + 1 = 0$. Give your answers in surd form.

.... **(2 marks)**

Simultaneous equations 1

Guided

1 Solve the simultaneous equations

(a) $2x + 5y = 16$ ①
$5x - 2y = 11$ ②

① × 5 gives

......x + ...y = ... ③

② × 2 gives

$10x - 4y = 22$ ④

③ − ④ gives

......y = ...

y = ...

Substitute y = ... in ①:

$2x + 5$ ...... = 16

x = ...

x = ..., y = ... **(3 marks)**

(b) $3x + 2y = 11$
$2x - 5y = 20$

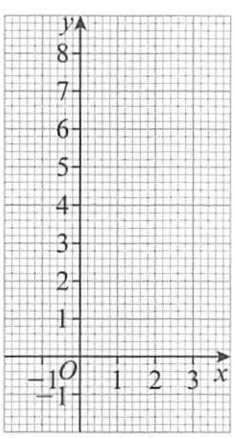
Label the equations ① and ②.

x = ..., y = ... **(3 marks)**

Guided

2 By drawing two suitable straight lines on the coordinate grids below, solve the simultaneous equations

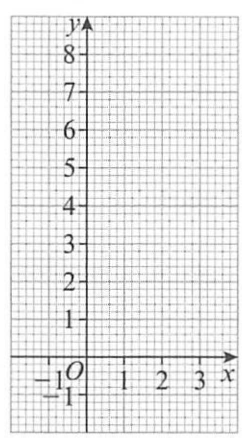
(a) $x + y = 5$
$y = 3x + 1$



x = ...,

y = ... **(4 marks)**

(b) $2x + y = 5$
$x + y = 3$



x = ...,

y = ... **(4 marks)**

For each equation choose three x-values and then find the corresponding y-values.

Plot these points and then draw a straight line through the three points. The solution to the simultaneous equations is the point where the lines cross.

3 A resistor costs r pence. A fuse costs f pence. Six resistors and two fuses cost 72p altogether.

(a) Show that $3r + f = 36$

(1 mark)

Five resistors and three fuses cost 80p altogether.

(b) Work out the cost of one resistor and the cost of one fuse.

Resistor = ... p

Fuse = ... p **(4 marks)**

Simultaneous equations 2

Guided

1 Solve the simultaneous equations
$y = x + 1$
$x^2 + y^2 = 5$

Always substitute the linear equation into the quadratic equation.

$x^2 + y^2 = 5$

$x^2 + (\ldots\ldots + \ldots\ldots)^2 = 5$

Multiply out the brackets and then solve the quadratic equation.

$x^2 + x^2 + \ldots\ldots x + \ldots\ldots = 5$

$2x^2 + \ldots\ldots x - \ldots\ldots = 0$

Cancel.

$x^2 + \ldots\ldots x - \ldots\ldots = 0$

Factorise the expression.

$(x + \ldots\ldots)(x - \ldots\ldots) = 0$

$x = \ldots\ldots$ and $x = \ldots\ldots$

Substitute these values into the linear equation.

$y = \ldots\ldots + 1 = \ldots\ldots$ and $y = \ldots\ldots + 1 = \ldots\ldots$ **(5 marks)**

2 Solve the simultaneous equations
$y = 6 - 2x$
$xy + x = 3$

.... **(5 marks)**

3 Solve the simultaneous equations
$x + y = 4$
$y = 2x^2 - 7x + 8$

Rearrange $x + y = 4$ to $y = \ldots$

.... **(5 marks)**

4 L is the straight line with equation $y = x - 3$
C is the circle with equation $x^2 + y^2 = 45$
The line intersects the circle at two points.
Find the coordinates of both points.

.... and .... **(5 marks)**

Equation of a circle

Guided

1 Write down the equation of a circle with

(a) centre (0, 0) and radius 6.

(b) centre (0, 0) and radius 13.

(c) centre (0, 0) and radius $\frac{5}{4}$.

$x^2 + y^2 = r^2$

$x^2 + y^2 =$ .... **(1 mark)** .... **(1 mark)** .... **(1 mark)**

2 (a) Solve the simultaneous equations

$y = x - 2$

$x^2 + y^2 = 2$

.... **(5 marks)**

(b) Use your answer to part (a) to state the geometrical relationship between the line $y = x - 2$ and the circle $x^2 + y^2 = 2$

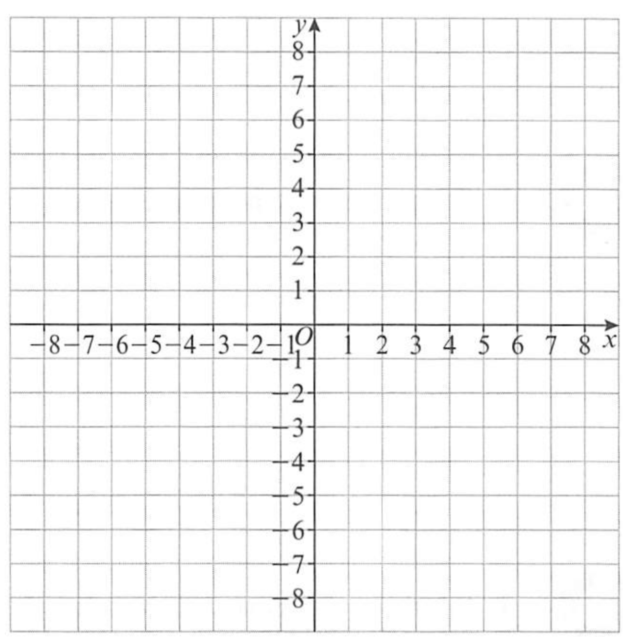
.... **(1 mark)**

3 (a) Draw the graph of $x^2 + y^2 = 25$. **(2 marks)**

(b) Draw the graph of $y = x + 2$ on the same axes. **(1 mark)**

(c) Use your graph to write down the solutions to the simultaneous equations $y = x + 2$ and $x^2 + y^2 = 25$.

.... **(2 marks)**



PROBLEM SOLVED!

4 **C** is the circle with equation $x^2 + y^2 = 52$

(a) Verify that the point (4, −6) lies on **C**.

You will need to use problem-solving skills throughout your exam – **be prepared!**

(1 mark)

(b) **T** is a tangent to the circle at the point (4, −6). Find the equation of **T**.

.... **(5 marks)**

Inequalities

1 Solve

(a) $2x - 3 \leqslant 5$ (+ 3)

Guided

$2x \leqslant 5 +$ ....

$2x \leqslant$ .... (÷ 2)

$x \leqslant$ .... **(2 marks)**

(b) $6x - 3 > 2x + 9$

.... **(2 marks)**

(c) $3 + x < 25 + 3x$

.... **(2 marks)**

2 Find the integer values of x that satisfy both of these inequalities.

Guided

(a) $x - 5 > 3$ and $3x + 1 < 31$

$x - 5 > 3$ (+ 5)

$x > 3 +$ ....

$x >$ ....

.... **(3 marks)**

(b) $2x + 1 > 7$ and $4x - 3 < 17$

.... **(3 marks)**

3 Find the integer values of x that satisfy both of these inequalities.

$3(x - 2) > x - 4$ and $4x + 12 < 2x + 18$

.... **(3 marks)**

4 Solve the inequalities

(a) $\frac{8x - 5}{3} \geqslant 9$

.... **(3 marks)**

(b) $\frac{2x + 3}{3} < \frac{3x + 5}{7}$

.... **(3 marks)**

5 A rectangular patio has length x m. The length is 7 m more than the width.
The perimeter of the patio must be less than 53 m.

(a) Form a linear inequality in x.

.... **(2 marks)**

(b) Solve your inequality to find the greatest value of x where x is an integer.

$x =$ .... **(1 mark)**

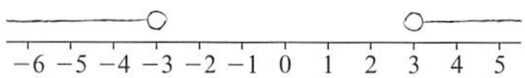
Quadratic inequalities

1 Solve these inequalities. Represent your solutions on the number line.

Guided

(a) $x^2 - 9 > 0$

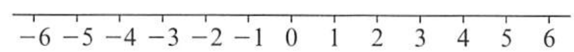
$(x + \ldots\ldots)(x - \ldots\ldots) > 0$



.... **(2 marks)**

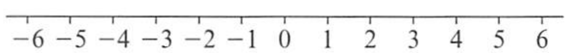
(b) $x^2 - 25 \geqslant 0$

Factorise the quadratic.



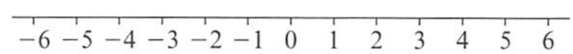
.... **(2 marks)**

(c) $x^2 \leqslant 16$ $\quad x^2 - 16 \leqslant 0$



.... **(2 marks)**

(d) $x^2 > 1$



.... **(2 marks)**

2 Solve these inequalities. Leave your answers in surd form.

(a) $x^2 - 27 < 0$

.... **(2 marks)**

(b) $x^2 - 48 \geqslant 0$

.... **(2 marks)**

3 Solve these inequalities.

(a) $x^2 - 4x - 5 < 0$

.... **(2 marks)**

You are interested in the values of x where the graph is below the horizontal axis.

Start by sketching the graph of $y = x^2 - 4x - 5$.

(b) $x^2 - 11x + 24 \geqslant 0$

.... **(2 marks)**

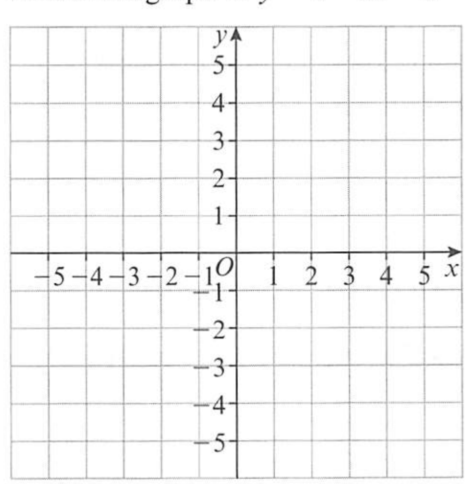
(c) $x^2 - 3x - 10 \leqslant 0$

.... **(2 marks)**

(d) $4x^2 - 8x + 3 > 0$

.... **(2 marks)**

4 (a) Sketch the graph of $y = 3 - 2x - x^2$



(2 marks)

(b) Hence, or otherwise, solve the inequality $3 - 2x - x^2 < 0$

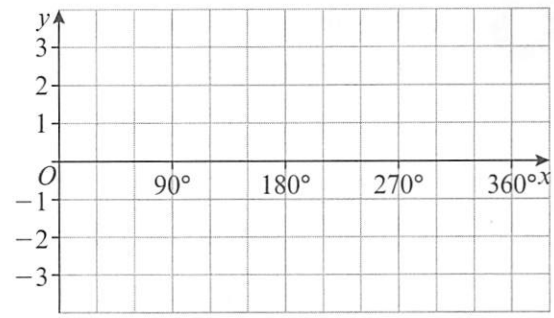
.... **(2 marks)**

Trigonometric graphs

1 (a) Sketch the graph of $y = \tan x$ in the range $0 \leqslant x \leqslant 360°$. **(2 marks)**

(b) State the equation of an asymptote.

.... **(2 marks)**



2 Find the two values of angle A, $0 \leqslant A \leqslant 360°$, such that

Guided

(a) $\sin A = 0.5$

Find the first angle.

$A = \sin^{-1} 0.5$

$A =$ ....°

Second angle = ...° − ....° = ... **(2 marks)**

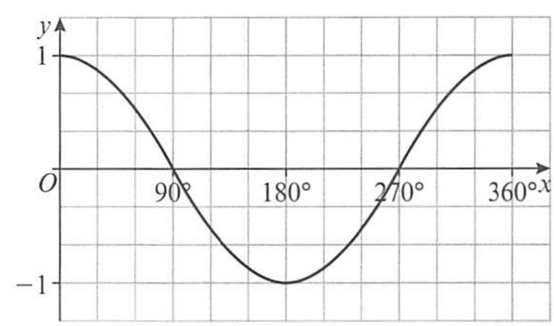
(b) $\cos A = \frac{\sqrt{3}}{2}$

.... **(2 marks)**

(c) $\tan A = 0.68$

.... **(2 marks)**

3 The diagram shows the graph of $y = \cos x$ for $0 \leqslant x \leqslant 360°$



(a) Mark on the diagram the two solutions of $\cos x = -0.5$ **(1 mark)**

(b) Write down the two solutions of $\cos x = -0.5$

.... **(2 marks)**

(c) State the coordinates of the turning points.

.... **(2 marks)**

4 (a) On the axes, sketch the graph of $y = \sin x$ for $0 \leqslant x \leqslant 360°$.
Label the axes and mark the scales clearly. **(3 marks)**

(b) Hence, or otherwise, state the two values of angle x, $0 \leqslant x \leqslant 360°$, such that $\sin x = \frac{\sqrt{3}}{2}$

.... **(2 marks)**

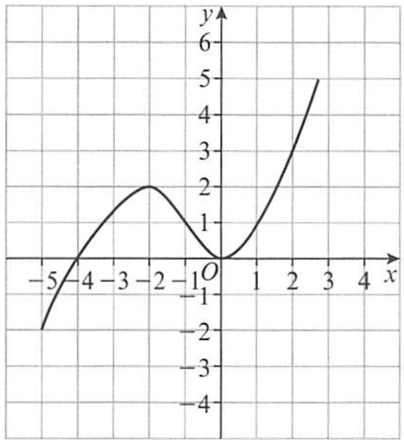
(c) State the coordinates of the turning points.

.... **(1 mark)**

Transforming graphs

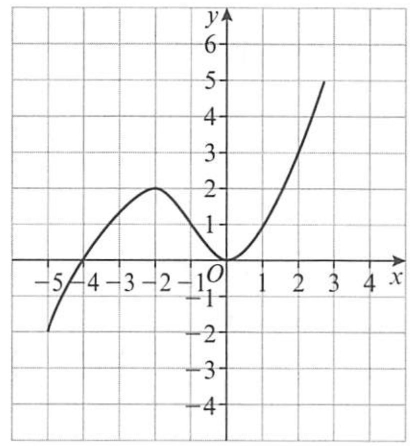
1 The graph of $y = f(x)$ is shown on each diagram. Sketch the graph of

(a) $y = f(x) + 2$



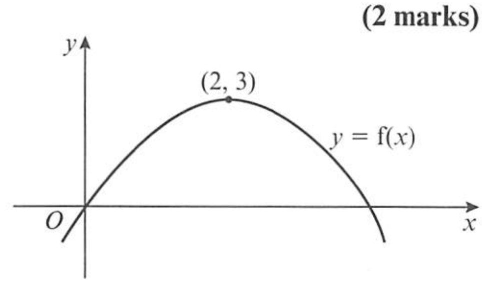
(2 marks)

(b) $y = -f(x)$



(2 marks)

2 The diagram shows part of the curve with equation $y = f(x)$
The coordinates of the maximum point of this curve are (2, 3).
Write down the coordinates of the maximum point of the curve with equation



(a) $y = f(x - 4)$

.... **(1 mark)**

(b) $y = f(x) + 4$

.... **(1 mark)**

(c) $y = -f(x)$

.... **(1 mark)**

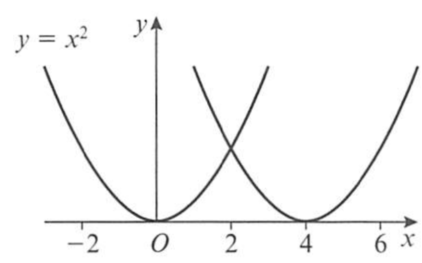
(d) $y = f(-x)$

.... **(1 mark)**

3 The curve with equation $y = x^2$ is translated so that the point at (0, 0) is mapped onto the point (4, 0). Find an equation of the translated curve.

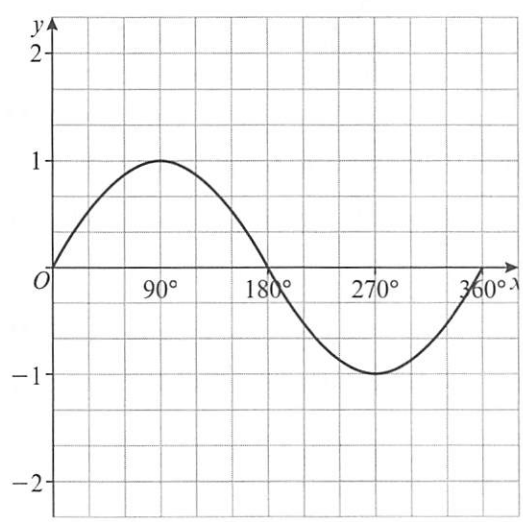
Guided

$y = (x$ ....$)^{....}$



(2 marks)

4 This graph shows the curve $y = \sin x$ for $0 \leqslant x \leqslant 360°$
On the same axes sketch the graph of $y = -\sin x$ for $0 \leqslant x \leqslant 360°$.



(2 marks)

Inequalities on graphs

1 (a) Shade in the region which satisfies these inequalities

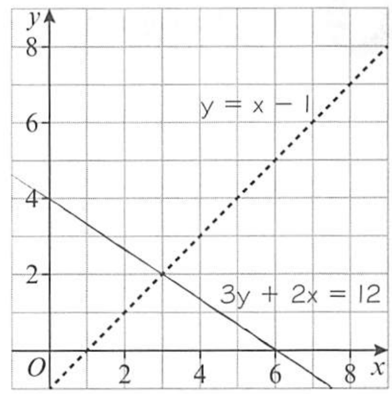
$3y + 2x \geqslant 12$ $y < x - 1$ $x < 6$

(2 marks)

Guided

Draw the graph of $x = 6$.

Use dotted lines (for < and >) and solid lines (for ⩽ and ⩾). Points on a solid line are included in the region but points on a dotted line aren't.



(b) x and y are integers. On the grid, mark with a cross (×) each of the **four** points which satisfies **all** three inequalities.

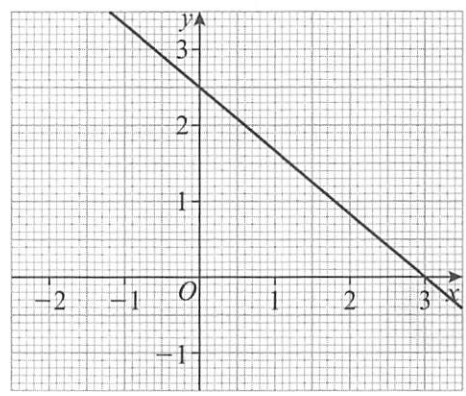
(2 marks)

2 The line with equation $6y + 5x = 15$ is drawn on the grid.

(a) (i) On the grid, shade the region of points whose coordinates satisfy these four inequalities.

$y \geqslant 0$ $x > 0$ $2x < 3$ $6y + 5x \leqslant 15$

(ii) Label this region **R**. **(2 marks)**



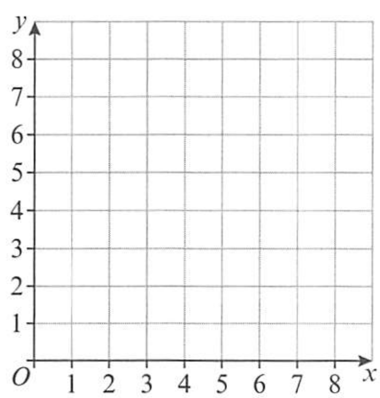
P is a point in the region **R**.
The coordinates of P are both integers.

(b) Write down the coordinates of P.

.... **(1 mark)**

3 (a) On the grid below, draw straight lines and use shading to show the region **R** that satisfies these inequalities.

$x > 2$ $y \leqslant x$ $x + y \leqslant 6$ $y > 0$



(3 marks)

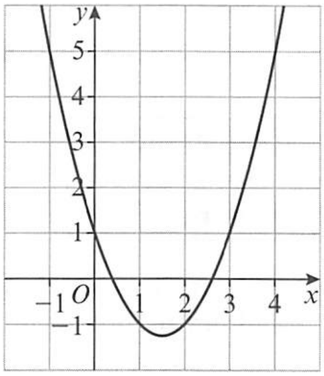
(b) The point P with coordinates (x, y) lies inside the region **R**. x and y are integers. Write down the coordinates of all the possible points of P.

.... **(2 marks)**

Had a go ☐ Nearly there ☐ Nailed it! ☐

Using quadratic graphs

1 The diagram shows a graph of $y = x^2 - 3x + 1$



(a) Find estimates for the solutions of $x^2 - 3x + 1 = 0$

.... **(1 mark)**

(b) Use a graphical method to find estimates for the solutions to the equation $x^2 - 3x + 1 = x - 1$

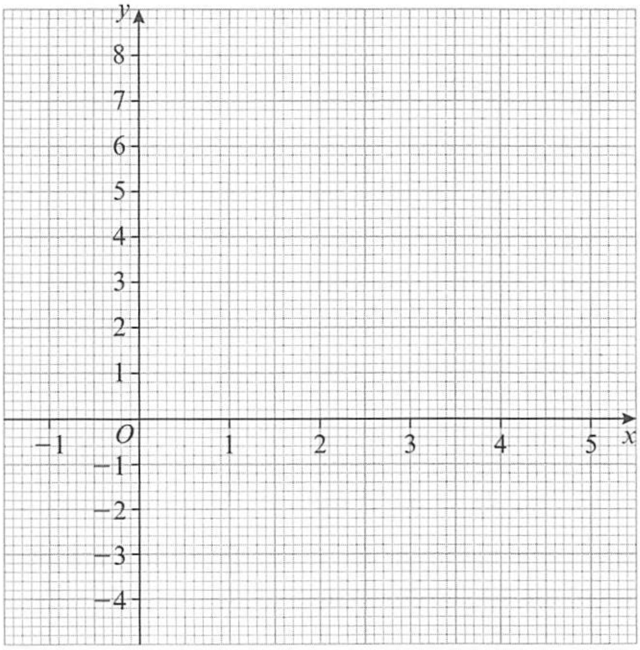
.... **(2 marks)**

2 (a) Complete the table of values for $y = x^2 - 4x$

x	−1	0	1	2	3	4	5
y		0	−3		−3		5

(2 marks)

(b) On the grid, draw the graph of $y = x^2 - 4x$



(2 marks)

(c) Find estimates for the solutions of $x^2 - 4x = 0$

.... **(2 marks)**

(d) By drawing a suitable straight line, work out the solutions of the equation $x^2 - 5x + 2 = 0$

.... **(2 marks)**

Turning points

1 Write down the coordinates of the turning points of the following curves.

(a) $y = x^2 + 4x - 6$

Guided

$y = (x + \ldots\ldots)^2 - \ldots\ldots - 6 = (x + \ldots\ldots)^2 - \ldots\ldots$

Turning point = (......., ...) **(3 marks)**

(b) $y = x^2 + 2x - 4$ (c) $y = x^2 + 3x + 2$ (d) $y = 2x^2 + 3x - 7$

.... **(3 marks)** .... **(3 marks)** .... **(3 marks)**

2 (a) By writing the quadratic equation $y = x^2 - 4x + 2$ in the form $(x + p)^2 + q$, find p and q.

p = ...

q = ... **(3 marks)**

(b) Hence, or otherwise, write down the minimum value of the equation.

.... **(1 mark)**

3 Sketch the following graphs of $y = f(x)$, showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.

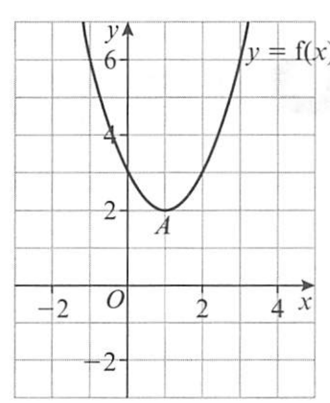
PROBLEM SOLVED!

(a) $f(x) = x^2 + 3x + 10$ (b) $f(x) = x^2 - 4x + 4$

You will need to use problem-solving skills throughout your exam – **be prepared!**

(4 marks) **(4 marks)**

4 The diagram shows a graph of the curve with equation $y = f(x)$



(a) Write down the coordinates of the turning point.

.... **(1 mark)**

The curve with equation $y = f(x)$ is a translation of the curve with equation $y = x^2$

(b) Find the equation of the curve $y = f(x)$.

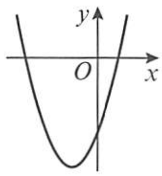
.... **(2 marks)**

Sketching graphs

Guided

1 Look at the graphs sketched below. Write down any intercepts with the coordinate axes.

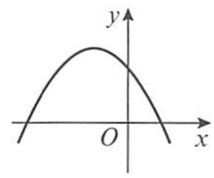
(a) $y = x^2 + 4x - 21$



$y = (x + \ldots\ldots)(x - \ldots\ldots)$

$x = \ldots\ldots$ and $x = \ldots\ldots$ **(2 marks)**

(b) $y = 3 - 5x - 2x^2$

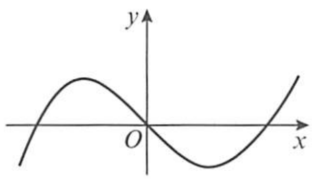


.... **(2 marks)**

Guided

2 Look at the graphs sketched below. Write down any intercepts with the coordinate axes.

(a) $y = x(x - 3)(x + 3)$

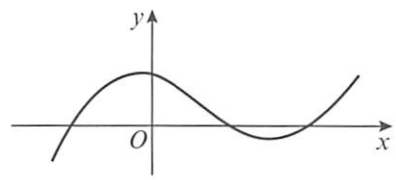


If one factor of a cubic equation is x, then $x = 0$ is one solution.

$x(x - 3)(x + 3) = 0$

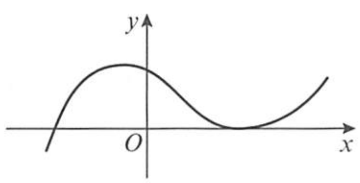
$x = \ldots\ldots$, $x = \ldots\ldots$ and $x = \ldots\ldots$ **(2 marks)**

(b) $y = (x - 4)(x - 2)(x + 5)$



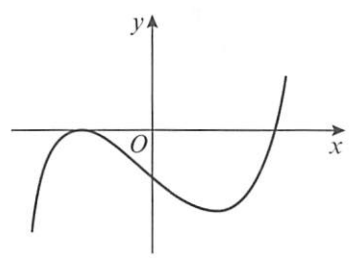
.... **(2 marks)**

(c) $y = (x - 1)^2(x + 2)$



.... **(2 marks)**

(d) $y = (x + 4)^2(x - 1)$



.... **(2 marks)**

3 Sketch the following graphs of $y = f(x)$, showing the coordinates of any intercepts with the coordinate axes.

(a) $y = x^2 - 8x + 15$

(2 marks)

(b) $y = x(x^2 + 3x - 18)$

(3 marks)

Iteration

1 (a) Show that when $f(x) = 0$, the equation $f(x) = x^3 + x - 3$ can be rearranged to give $x = \sqrt[3]{3 - x}$

Guided

$x^3 + x - 3 = 0$

Rearrange the equation for x^3

$x^3 = ... - ....$

Take the cube root of the right-hand side.

$x = \sqrt[3]{....... - ....}$

(2 marks)

(b) Use the iterative formula $x_{n+1} = \sqrt[3]{3 - x_n}$ with $x_0 = 1$ to find the real root of $f(x)$ correct to 2 decimal places.

Work out x_1, x_2, x_3 and so on. Write down all the digits shown on your calculator display, then round each value to 2 d.p. until you reach two values that both round to the same number.

.... **(3 marks)**

2 (a) Show that when $f(x) = 0$, the equation $f(x) = x^3 - 7x + 2$ can be rearranged to give $x = \sqrt[3]{7x - 2}$

.... **(1 mark)**

(b) Use the iterative formula $x_{n+1} = \sqrt[3]{7x_n - 2}$ with $x_0 = -2.4$ to find the real root of $f(x)$ correct to 3 decimal places.

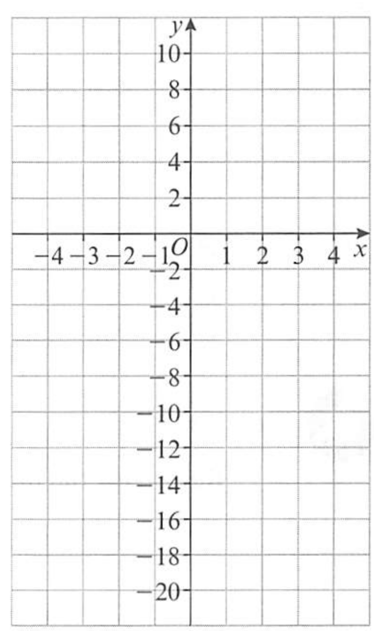
.... **(3 marks)**

3 (a) Complete the table of values for $y = x^3 - 5x - 8$

x	−3	−2	−1	0	1	2	3
y	−20			−8	−12		4

(2 marks)

(b) On the grid, draw the graph of $y = x^3 - 5x - 8$ **(2 marks)**



(c) Show that when $f(x) = 0$, the equation $f(x) = x^3 - 5x - 8$ can be rearranged to give $x = \sqrt[3]{5x + 8}$

(2 marks)

(d) Use the iterative formula $x_{n+1} = \sqrt[3]{5x_n + 8}$ to find the real root of $f(x)$ correct to 3 decimal places.

.... **(3 marks)**

Had a go ☐ Nearly there ☐ Nailed it! ☐

Rearranging formulae

1 Rearrange $y = \frac{1}{3}x + 4$ to make x the subject.

Guided

$y = \frac{1}{3}x + 4$ (− 4)

$y - \ldots\ldots = \frac{1}{3}x$ (× 3)

$\ldots\ldots(y - \ldots\ldots) = x$

Insert brackets around the expression on the left-hand side.

(2 marks)

2 Make h the subject of the formula $d = \sqrt{\frac{3h}{2}}$

Guided

$d = \sqrt{\frac{3h}{2}}$ (square both sides)

$d^{\ldots} = \frac{3h}{2}$ (× 2)

$\ldots\ldots d^{\ldots} = 3h$ (÷ 3)

$h = \ldots\ldots d^{\ldots}$

(2 marks)

3 Rearrange the formula $P = \frac{3w + 10}{120}$ to make w the subject.

.... **(2 marks)**

4 Rearrange the formula $P = \pi r + 2r + 2x$ to make x the subject.

.... **(2 marks)**

5 $M = \frac{t^2 + x}{t + x}$

Guided

Rearrange the formula to make x the subject.

$M = \frac{t^2 + x}{t + x}$ [× $(t + x)$]

Multiply both sides by the denominator.

$M(t + x) = t^2 + x$ (expand brackets)

$M\ldots\ldots + M\ldots\ldots = t^2 + x$ ($-t^2$ and $-Mx$)

$M\ldots\ldots - \ldots\ldots = x - \ldots\ldots$

Collect all the x terms on one side.

$M\ldots\ldots - \ldots\ldots = x(\ldots\ldots - \ldots\ldots)$

Factorise the right-hand side.

$\frac{M\ldots\ldots - \ldots\ldots}{\ldots\ldots - \ldots\ldots} = x$

(3 marks)

6 $y = \frac{2pt}{p - t}$

Rearrange the formula to make t the subject.

.... **(3 marks)**

Algebraic fractions

Guided

1 Simplify fully

(a) $\frac{3}{4x} + \frac{1}{8x}$

Find the common denominator.

$\frac{\dots\dots}{\dots\dots x} + \frac{\dots\dots}{\dots\dots x} = \frac{\dots\dots + \dots\dots}{\dots\dots x} = \frac{\dots\dots}{\dots\dots x}$

(2 marks)

(b) $\frac{x+1}{2} - \frac{x-2}{3}$

.... **(2 marks)**

(c) $\frac{2}{x+3} - \frac{1}{x-2}$

.... **(2 marks)**

(d) $\frac{x}{y} + \frac{y}{x}$

.... **(2 marks)**

Guided

2 Simplify fully

(a) $\frac{x^2 + 2x + 1}{4x + 4}$

Factorise the numerator and the denominator and then cancel.

$\frac{(x + \dots\dots)(x + \dots\dots)}{\dots\dots(x + \dots\dots)} = \frac{(x + \dots\dots)}{\dots\dots}$

(2 marks)

(b) $\frac{x^2 + 4x - 5}{x^2 + 2x - 3}$

.... **(2 marks)**

Guided

3 Simplify fully

(a) $\frac{3}{x} \times \frac{x}{5}$

Cancel first.

$\frac{3}{\cancel{x}} \times \frac{\cancel{x}}{5} = \dots\dots\dots\dots$ **(2 marks)**

(b) $\frac{2r^2}{5} \times \frac{4}{r^3}$

.... **(2 marks)**

(c) $\frac{5}{xy} \div \frac{x}{y}$

.... **(2 marks)**

(d) $\frac{x+2}{x+4} \div \frac{3x+6}{x^2-16}$

.... **(2 marks)**

4 Solve

(a) $\frac{3x}{4} + \frac{2}{3} = x$

$x =$ .... **(3 marks)**

(b) $\frac{3}{4x} - \frac{1}{2x} = 4$

$x =$ .... **(3 marks)**

(c) $\frac{3x+2}{2} - \frac{x-1}{5} = 3$

$x =$ .... **(3 marks)**

(d) $\frac{5}{x+1} - \frac{3}{2(x+1)} = \frac{1}{2}$

.... **(3 marks)**

Had a go ☐ Nearly there ☐ Nailed it! ☐

Quadratics and fractions

1 Solve

Guided

(a) $\frac{3}{x} + \frac{1}{x-4} = 1$

Multiply everything by $x(x - 4)$ to remove the fractions.

$\frac{3 \ldots\ldots(\ldots\ldots - \ldots\ldots)}{x} + \frac{1 \ldots\ldots(\ldots\ldots - \ldots\ldots)}{x-4} = 1 \ldots\ldots(\ldots\ldots - \ldots\ldots)$

Cancel.

$3(\ldots\ldots - \ldots\ldots) + 1 \ldots\ldots = x(\ldots\ldots - \ldots\ldots)$

Multiply out and simplify.

$\ldots\ldots x - \ldots\ldots + 1x = x^{\ldots} - \ldots\ldots$

Rearrange as a quadratic equation.

$x^{\ldots} - \ldots\ldots x + \ldots\ldots = 0$

Solve the quadratic equation using factorisation.

$(x\ldots\ldots)(x\ldots\ldots) = 0$

$x = \ldots\ldots$ and $x = \ldots\ldots$

(4 marks)

(b) $\frac{2}{x-3} + \frac{1}{x-4} = 2$

.... **(4 marks)**

(c) $\frac{2}{x+1} - \frac{3}{2x+3} = 1$

.... **(4 marks)**

(d) $\frac{3}{2x-1} - \frac{2}{1-3x} = 4$

.... **(4 marks)**

(e) $\frac{1}{x} - \frac{2}{x-1} = 6$

.... **(4 marks)**

(f) $\frac{x+1}{4} - \frac{20}{x-5} = 1$

.... **(4 marks)**

(g) $\frac{x+4}{x-2} = x$

.... **(4 marks)**

2 Jack travels a distance of 400 km. He travels at an average speed of x km/h.

(a) Write down an expression in terms of x for the time Jack travels.

.... **(1 mark)**

(b) Jack increases his average speed by 10 km/h for his return journey.
Write down an expression in terms of x for the time taken for his return journey.

.... **(2 marks)**

(c) If Jack's travelling time is 40 minutes less at the faster average speed, show that
$\frac{400}{x} - \frac{400}{x+10} = \frac{2}{3}$

.... **(2 marks)**

(d) Work out the value of x. Give your answer correct to 3 significant figures.

.... **(2 marks)**

Surds 2

1 Show that $(4 + \sqrt{3})^2 = 19 + 8\sqrt{3}$
Show each stage of your working clearly.

Guided

When the question says 'Show that …' you should start from the left-hand side, then simplify and rearrange until your expression matches the right-hand side.

$(4 + \sqrt{3})(4 + \sqrt{3}) =$ .... + ...$\sqrt{3}$ + ...$\sqrt{3}$ + ...

= .... + ...$\sqrt{3}$

(3 marks)

2 Show that $(\sqrt{2} + \sqrt{3})^2 = 5 + 2\sqrt{6}$
Show each stage of your working clearly.

(3 marks)

3 Show that $(4 + \sqrt{5})(2 - \sqrt{5}) = 3 - 2\sqrt{5}$
Show each stage of your working clearly.

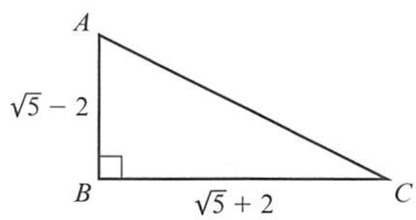
(3 marks)

4 a and b are positive integers such that $(1 - \sqrt{a})^2 = b - 2\sqrt{7}$
Find the value of a and the value of b.

$a =$ ....

$b =$ .... **(3 marks)**

5 The diagram shows a right-angled triangle. All measurements are in cm.



Work out, leaving your answers in surd form where appropriate,

(a) the area of the triangle

.... cm² **(3 marks)**

(b) the length of AC.

.... cm **(3 marks)**

Functions

1 f and g are functions such that $f(x) = 2x - 1$ and $g(x) = \frac{2}{x}, x \neq 0$

(a) Find the value of

Guided

(i) f(7)

Substitute $x = 7$ into f(x) and simplify.

f(7) = 2....... − 1 = ...

(1 mark)

(ii) fg(3)

Order is important. Substitute $x = 3$ into g(x) first and then substitute this answer into f(x)

.... **(1 mark)**

(b) Find gf(x)

Substitute $2x - 1$ into g(x)

$g(x) = \frac{2}{\ldots\ldots\ldots\ldots}$

(2 marks)

2 f and g are functions. $f(x) = 4x - 3$ $g(x) = 1 + x^3$

(a) Find f(−2)

.... **(1 mark)**

(b) Given that $g(a) = 28$, find the value of a.

.... **(2 marks)**

(c) Work out gf(2)

.... **(2 marks)**

(d) Work out fg(x)

.... **(2 marks)**

3 The function f is defined as $f(x) = \frac{x}{x - 2}, x \neq 2$

(a) Find the value of f(3)

.... **(1 mark)**

(b) Find ff(x). Give your answer in its simplest form.

.... **(3 marks)**

4 f and g are functions such that $f(x) = x^2$ and $g(x) = 3x - 2$
Solve $fg(x) = f(x)$

.... **(4 marks)**

Inverse functions

1 Find the inverse function, f^{-1}, of the function

Guided

(a) $f(x) = 3x - 5$

Make the x the subject.

Let $y = 3x - 5$

$3x = y + ...$

$x = \frac{y + ...}{.......}$

Therefore, $f^{-1} =$ .... **(2 marks)**

(b) $f(x) = \frac{4x}{3} + 7$

f^{-1} .... **(2 marks)**

(c) $f(x) = \frac{x - 7}{x}$

f^{-1} .... **(2 marks)**

2 (a) $f(x) = 3x + 4$

Find $f^{-1}(x)$

$f^{-1}(x)$ .... **(2 marks)**

(b) $g(x) = 4x - 1$

Find $g^{-1}(x)$

$g^{-1}(x)$ .... **(2 marks)**

(c) Hence, or otherwise, find an expression for $f^{-1}(x) + g^{-1}(x)$
You must simplify your answer.

.... **(2 marks)**

3 The function f is such that $f(x) = 2x + 1$

(a) Find $f^{-1}(x)$

$f^{-1}(x)$ .... **(2 marks)**

(b) State the value of $ff^{-1}(5)$

$ff^{-1}(x)$ .... **(1 mark)**

4 The function g is such that $g(x) = \frac{x}{x - 1}$

(a) Solve the equation $g(x) = \frac{5}{2}$

$x =$ .... **(2 marks)**

(b) Find $g^{-1}(x)$

$g^{-1}(x)$ .... **(3 marks)**

Algebraic proof

Guided

1 Prove that $(2n - 1)^2 + (2n + 1)^2 = 2(4n^2 + 1)$

Simplify by multiplying out the brackets.

LHS $= (2n - 1)^2 + (2n + 1)^2$

$= (2n - 1)(2n - 1) + (2n + 1)(2n + 1)$

Add the brackets together and then factorise.

(2 marks)

2 $5(x - c) = 4x - 5$ where c is an integer
Prove that x is a multiple of 5.

Multiply out the brackets, then rearrange to make x the subject.

(3 marks)

3 Prove that $(3x + 1)^2 - (3x - 1)^2$ is a multiple of 4, for all positive integer values of x.

(3 marks)

PROBLEM SOLVED!

4 Prove that the sum of any three consecutive even numbers is always a multiple of 6.

You will need to use problem-solving skills throughout your exam – **be prepared!**

Write your even numbers as $2n$, $2n + 2$ and $2n + 4$

(3 marks)

5 A rectangular number is defined as $n(n + 1)$. Prove that the sum of two consecutive rectangular numbers is always double a square number.

(3 marks)

6 The nth term of a sequence is given by $x_n = \frac{1}{n}$

Prove that $x_n - x_{n+1} = \frac{1}{n(n + 1)}$

(3 marks)

7 Prove that the difference between the square of any 2-digit number and the square of that number when reversed has a common factor of 99.

For example:
$73^2 - 37^2 = 3960$
$= 99 \times 40$

(5 marks)

Exponential graphs

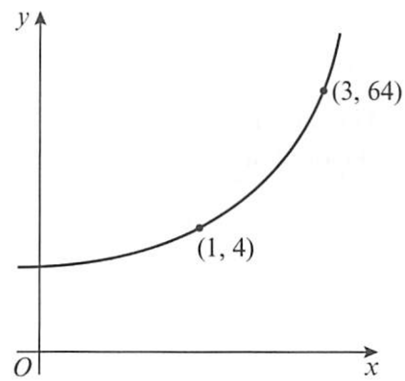
1 Sketch the graph of

(a) $y = 3^x$ (b) $y = 5^x$ (c) $y = 2^{-x}$

(2 marks) **(2 marks)** **(2 marks)**

2 The sketch shows a curve with equation $y = ka^x$ where k and a are constants, and $a > 0$.
Show that the value of a is 4 times the value of k.

Guided



Substitute (1, 4) and (3, 64) into the equation $y = ka^x$.

.... = $ka^{....}$ and .... = $ka^{....}$

Solve the equations simultaneously by dividing the second equation by the first.

(4 marks)

3 A scientist is investigating the growth in population of a certain bacteria. At 12 noon, there are 2000 bacteria in a Petri dish. The population of bacteria grows exponentially at the rate of 20% per hour.

(a) Show that the population after 3 hours is 3456.

(2 marks)

(b) The number of bacteria, N, in the Petri dish h hours after 12 noon can be modelled by the formula $N = a \times b^h$ where a and b are to be determined.
Write down the value of a and the value of b.

$a =$ ....

$b =$ .... **(2 marks)**

The population of bacteria in the Petri dish after 12 hours is k times the population of the bacteria after 9 hours.

(c) Find the value of k.

$k =$ .... **(2 marks)**

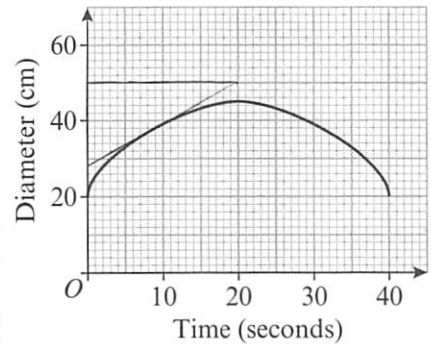
Had a go ☐ Nearly there ☐ Nailed it! ☐

Gradients of curves

Guided

1 A fish tank is filled with water. The graph shows how the diameter of the surface of the water changes with time.

Estimate the gradient when the time is 10 seconds.



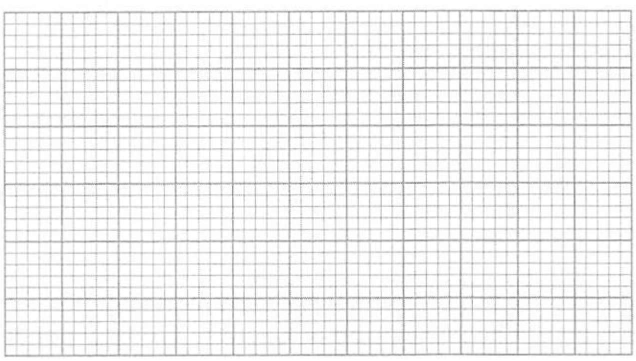
> The gradient of the curve at any point is the gradient of the tangent at that point. Draw a tangent at $t = 10$. Complete the triangle and work out the horizontal distance and the vertical distance.

gradient of tangent $= \dfrac{\text{vertical distance}}{\text{horizontal distance}} = \dfrac{\ldots\ldots - \ldots\ldots}{\ldots\ldots - \ldots\ldots} = \ldots\ldots\ldots\ldots\ldots\ldots$ **(2 marks)**

2 A tank is emptied and the depth of water is recorded. Here are the results.

Time, *t* (min)	0	1	2	3	4	5	6	7	8	9	10	11
Depth, *d* (m)	6.0	5.9	5.8	5.6	5.4	5.2	4.9	4.5	3.9	3.0	1.8	0.0

(a) Plot the graph of depth against time.



(2 marks)

(b) Work out the average rate of decrease of depth between $t = 0$ and $t = 11$.

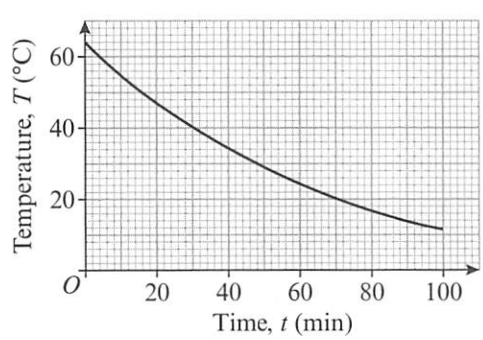
.... m/min **(2 marks)**

(c) Estimate the average rate of decrease of depth when the depth is 5 m.

.... m/min **(2 marks)**

3 In an experiment, Michaela heated a liquid to 64 °C then allowed it to cool.
The graph shows her results.

(a) Work out the average rate of decrease of temperature between $t = 0$ and $t = 100$.



.... °C/min **(2 marks)**

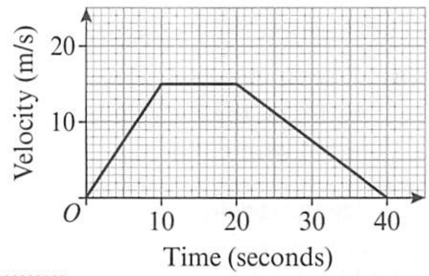
(b) Estimate the average rate of decrease of temperature at $t = 40$.

.... °C/min **(2 marks)**

Velocity–time graphs

Guided

1 The velocity–time graph shows the velocity of a car for the first 40 seconds of its journey.



(a) Work out the acceleration of the car for the first 10 seconds.

acceleration = $\frac{\text{change in velocity}}{\text{change in time}}$ = ...

= .... m/s^2 **(2 marks)**

(b) Work out the acceleration of the car for the last 20 seconds.

The acceleration is negative.

acceleration = $\frac{\text{change in velocity}}{\text{change in time}}$ = ...

= .... m/s^2 **(2 marks)**

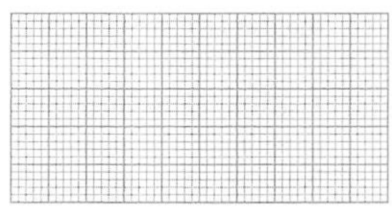
(c) Work out the total distance travelled in the 40 seconds.

Use the formula for the area of a trapezium, $A = \frac{1}{2}(a + b)h$

.... m **(3 marks)**

PROBLEM SOLVED!

2 A train accelerates uniformly from rest to 10 m/s in 20 seconds, then travels at a constant speed for 2 minutes and then decelerates at a constant rate to rest in 40 seconds.



(a) Sketch the velocity–time graph of the train's journey.

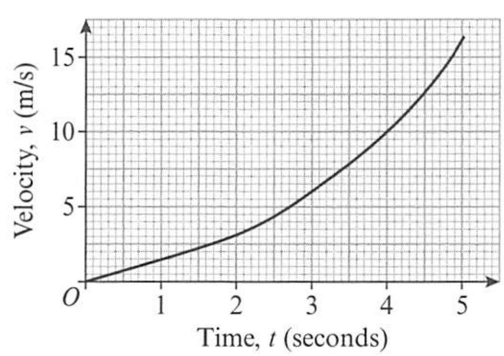
You will need to use problem-solving skills throughout your exam – **be prepared!**

(2 marks)

(b) Work out the total distance travelled.

.... m **(3 marks)**

3 The graph shows the velocity of an object for the first 5 seconds of its journey.



(a) Estimate the gradient at $t = 3$ seconds.

.... **(3 marks)**

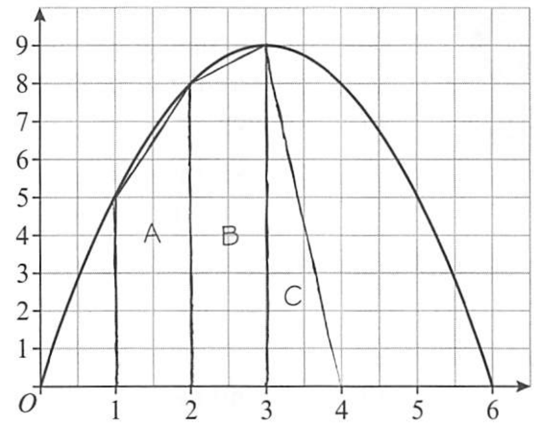
(b) Interpret your result from part (a).

.... **(2 marks)**

Areas under curves

Guided

1 The diagram shows the graph of $y = 6x - x^2$



(a) Use three equal intervals on the graph to estimate the area under the curve between $x = 1$ and $x = 4$.

Divide the required area into three trapeziums, work out the area of each trapezium, and add them to find the total.

Area of trapezium $= \frac{1}{2}(a + b)h$

Area of A $= \frac{1}{2}$(....... + ...)....... = ...

Area of B = ...

Area of C = ...

Total area = ... **(4 marks)**

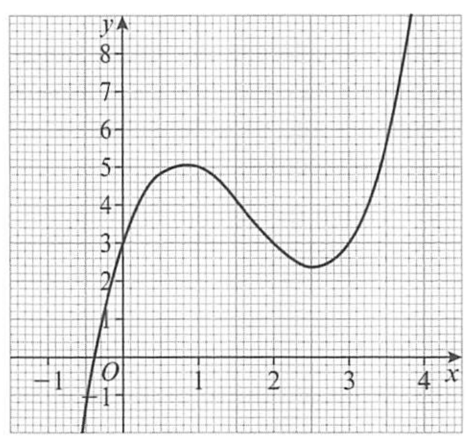
(b) Is your answer to part (a) an overestimate or an underestimate? Justify your answer.

Look at the shape of the curve.

The area is an .... because .... **(1 mark)**

2 The diagram shows the graph of $y = x^3 - 5x^2 + 6x + 3$

(a) Use four equal intervals on the graph to estimate the area under the curve between $x = 1$ and $x = 3$.



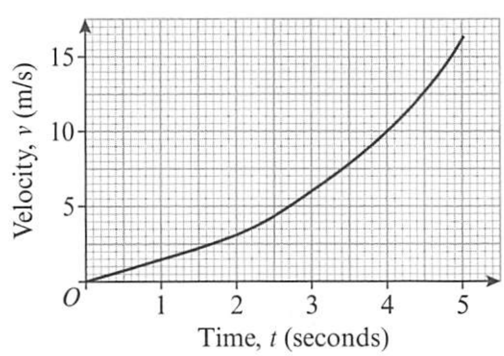
.... **(4 marks)**

(b) Is your answer to part (a) an overestimate or an underestimate? Justify your answer.

.... **(1 mark)**

3 The graph shows the velocity of an object for the first 5 seconds of its journey.

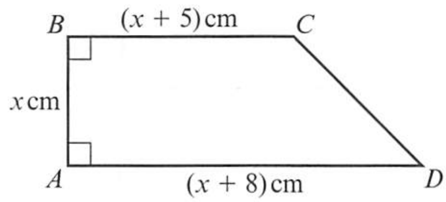
Use five equal intervals on the graph to estimate the distance travelled by the object in the first 5 seconds.



.... m **(4 marks)**

Problem-solving practice 1

1 The diagram shows a trapezium $ABCD$ with AD parallel to BC.



$AB = x$ cm $\quad BC = (x + 5)$ cm $\quad AD = (x + 8)$ cm

The area of the trapezium is 42 cm^2.

Show that x is a square number.

(4 marks)

2 x and y are positive integers such that $(3 - \sqrt{x})^2 = y - 6\sqrt{5}$

Work out the value of x and the value of y.

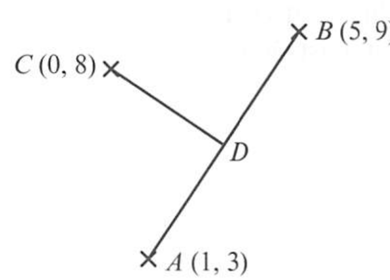
$x =$ ....

$y =$ .... **(3 marks)**

3 The diagram shows two straight lines, AB and CD. The coordinates of points A, B and C are $A(1, 3)$, $B(5, 9)$ and $C(0, 8)$. Point D is on AB and is halfway between point A and point B.

Marcie says that line CD is not perpendicular to line AB. Is she correct?

You must show all your working.



(4 marks)

4 Here are the first five terms of a quadratic series.

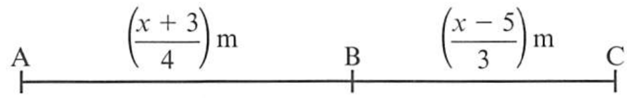
1 1 3 7 13

Work out an expression, in terms of n, for the nth term of this sequence.

.... **(4 marks)**

Problem-solving practice 2

5 The diagram shows two rods, AB and BC, joined together.



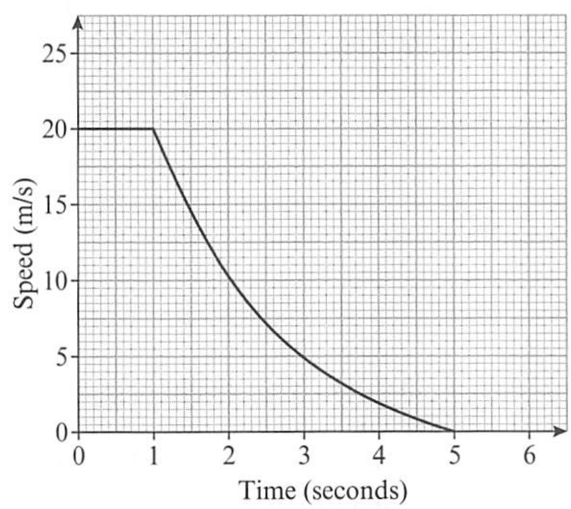
Rod AB is $\left(\frac{x+3}{4}\right)$ m in length and rod BC is $\left(\frac{x-5}{3}\right)$ m.

The total length of the rods is 9 m.

Work out the difference, in m, between the lengths of rod AB and rod BC.

.... m **(4 marks)**

6 A car approaches a set of traffic lights. The traffic lights turn red. The driver brakes and the car slows down to a stop. The distance, in metres, from the time the driver brakes to the traffic lights is 40 m. Here is the speed–time graph for the 5 seconds until the car stops. Does the car stop before the traffic lights or after the traffic lights? You must show your working.



(4 marks)

7 Find the coordinates of the points where line $y + 5 = 3x$ intersects the circle $x^2 + y^2 = 65$

.... **(5 marks)**

5 $\dfrac{1}{1+\frac{1}{\sqrt{3}}} = \dfrac{1}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \dfrac{\sqrt{3}}{\sqrt{3}+1}$

$= \dfrac{\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \dfrac{3-\sqrt{3}}{3-1} = \dfrac{3-\sqrt{3}}{2}$

13. Counting strategies

1 WX, WY, WZ, XY, XZ, YZ
2 (A,B) (A,C) (A,D) (B,C) (B,D) (C,D) 6 games
3 9
4 Yes, there are 1 757 600 different possible codes
5 (a) (i) 10 000 (ii) 28 561
(b) There are 900 possible codes

14. Problem-solving practice 1

1 60
2 (a) 3 packs of bread rolls, 5 packs of sausages, 8 packs of ketchup
(b) 120
3 (a) 1000 (b) 720
(c) $8^6 = 262\,144$ so 260 000 possible codes

15. Problem-solving practice 2

4 (a) 2.5×10^{25}
(b) Underestimate, because you are dividing by a number that has been rounded up
5 3.1, because both values of UB and LB are the same for 2 s.f.
6 $6\sqrt{2} + 14$

ALBEBRA

16. Algebraic expressions

1 (a) m^3 (b) d^4 (c) e^5
2 (a) x^{11} (b) y^5 (c) t^4
3 (a) x^6 (b) y^{15} (c) t^{21}
4 (a) x^5 (b) y^9 (c) t^6
5 (a) $28x^3y^7$ (b) $2x^3y$ (c) $81x^8y^{20}z^{12}$
6 (a) $5x^3$ (b) $64x^{4.5}y^6$ (c) $3x^{1.25}y^{0.75}$
7 (a) $9x^8$ (b) $\dfrac{8x^2y^5}{5}$ (c) $\dfrac{16x^2y^6}{9}$

17. Expanding brackets

1 (a) $x^2 + 7x + 12$ (b) $x^2 + 2x - 15$ (c) $x^2 - 8x + 12$
2 (a) $x^2 + 6x + 9$ (b) $x^2 - 8x + 16$ (c) $4x^2 + 4x + 1$
3 (a) $x^3 + 8x^2 + 15x$ (b) $x^3 + 2x^2 - 8x$ (c) $x^3 - 10x^2 + 21x$
4 (a) $x^3 + 9x^2 + 27x + 27$
(b) $x^3 - 12x^2 + 48x - 64$
(c) $8x^3 + 12x^2 + 6x + 1$
5 (a) $6x^2 + 12x - 20$ (b) $x^3 + 3x^2 - 10x - 24$

18 Factorising

1 (a) $3(x + 2)$ (b) $2(p - 3)$ (c) $5(y - 3)$
2 (a) $x(x + 6)$ (b) $x(x + 4)$ (c) $x(x - 12)$
3 (a) $3p(p + 2)$ (b) $8y(y - 3)$
4 (a) $4d(d + 3)$ (b) $6x(x - 3)$
5 (a) $(x + 3)(x + 1)$ (b) $(x + 10)(x + 1)$
6 (a) $(x + 7)(x - 1)$ (b) $(x + 5)(x - 1)$ (c) $(x - 5)(x + 3)$
7 (a) $(x - 3)(x + 3)$ (b) $(x - 12)(x + 12)$
8 (a) $(3x - 1)(x - 2)$ (b) $(2x - 3)(x + 1)$ (c) $(3x + 2)(x - 6)$

19. Linear equations 1

1 (a) 4 (b) 6 (c) 5
(d) 4 (e) 60 (f) −18
2 (a) 3 (b) 2 (c) −2
(d) $-\frac{3}{5}$ (e) 1 (f) $-\frac{5}{4}$
3 (a) 4 (b) −7 (c) 12
(d) 4 (e) 13 (f) 2
4 12

20. Linear equations 2

1 (a) $\frac{14}{3}$ (b) 4 (c) 3
2 (a) 1 (b) 12
3 (a) $\frac{55}{4}$ (b) 3
4 34

21. Formulae

1 (a) 8 (b) −19
2 (a) 23 (b) $\frac{35}{2}$
3 (a) 3500 (b) −200 (c) 8.84 (d) 19.8
4 26 000
5 $A = 2\pi r^2 + \dfrac{2V}{r}$

22. Arithmetic sequences

1 (a) $4n + 1$ (b) $3n - 1$ (c) $7n - 5$
2 $3n + 1$
3 $5n + 1$
4 (a) $4n - 1$ (b) Yes, because $n = 50$
5 (a) $6n - 3$
(b) No, because n is not a whole number

23. Solving sequence problems

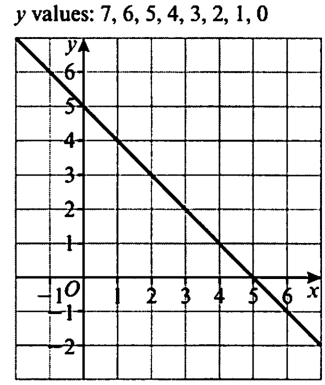
1 (a) $a = 3$ and $b = -1$
2 (a) 9 (b) $U_{n+1} = 2U_n + 3$
3 39
4 (a) 55 (b) $2x + 3y$ (c) $x = 5$ and $y = 6$
5 $27\sqrt{3}$, 81

24. Quadratic sequences

1 2, 7, 14, 23, 34, 47
2 (a) $n^2 + 2$ (b) $2n^2 + 4n - 3$
(c) $2n^2 + 2$ (d) $3n^2 - 2n + 1$
(e) $2n^2 + 4n - 4$ (f) $3n^2 - 5n + 3$
3 (a) $2n^2$
(b) No; if $2n^2 = 75$ then n is not a whole number
4 $n^2 + 3$

25. Straight-line graphs 1

1 y values: 7, 6, 5, 4, 3, 2, 1, 0



2 $y = 2x + 4$
3 $y = -\frac{20}{9}x + \frac{200}{9}$

26. Straight-line graphs 2

1 (a) $y = 3x - 1$
(b) $y = -2x + 12$
(c) $y = 4x + 15$
(d) $y = 4x - 2$
2 (a) $y = 2x - 4$
(b) $y = 2x + 1$
(c) $y = 2x$
3 $y = 4x + 5$
4 $k = 5$

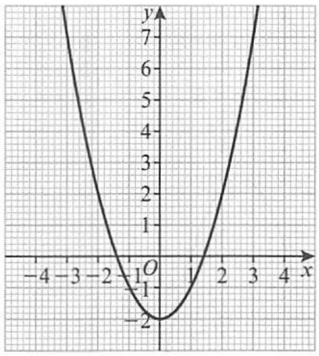
27. Parallel and perpendicular

1 (a) R (b) S
2 (a) $y = 3x - 7$ (b) $y = 4 - \frac{1}{3}x$
3 $y = -\frac{1}{3}x + 8$
4 (a) $y = 2x + 6$ (b) $y = -\frac{1}{2}x + 6$
5 $y = -2x$

28. Quadratic graphs

1 (a) y values: 7, 2, −1, −2, −1, 2, 7

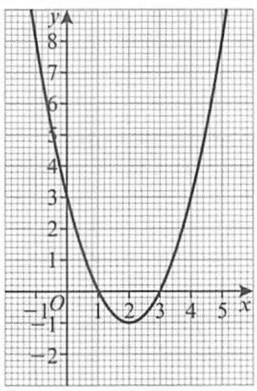
(b)



(c) (0, −2)

2 (a) y values: 8, 3, 0, −1, 0, 3, 8

(b)

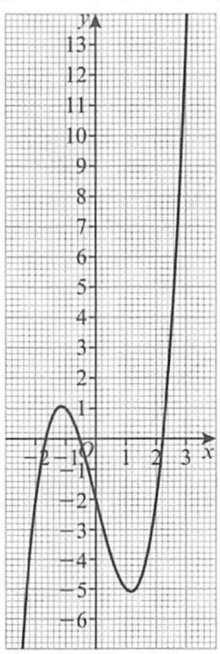


(c) (2, −1)

29. Cubic and reciprocal graphs

1 (a) y values: −2, 1, −2, −5, −2, 13

(b)



(c) (1.15, −5.08) and (−1.15, 1.08)

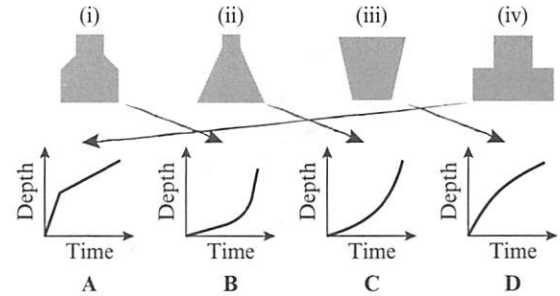
(d) −2.2, 0.3 and 1.9

2 (a) B (b) D (c) A (d) C (e) E

30. Real-life graphs

1 (a) 96 (b) Marseille

2



3 (a) −4

(b) Depth of water decreases by 4 cm every hour

31. Quadratic equations

1 (a) 0 or 3 (b) 0 or $-\frac{5}{2}$ (c) 0 or $\frac{7}{4}$

2 (a) 2 or −2 (b) $\frac{7}{4}$ or $-\frac{7}{4}$ (c) 5 or −5

3 (a) −2 or −4 (b) 3 or 4 (c) 3 or 7

4 (a) $-\frac{1}{2}$ or −3 (b) −1 or $-\frac{2}{3}$

5 (a) $(x + 5)(x - 2) = 60$

$x^2 + 3x - 70 = 0$

(b) length 12 cm, width 5 cm

6 1 or 2.5

32. The quadratic formula

1 (a) 1 (b) 2 (c) 0

2 (a) 4.56 or 0.438 (b) 0.281 or −1.78

(c) 1.15 or −0.348 (d) −3.56 or 0.562

3 (a) $x(2x + 1) + 5x = 95$

$2x^2 + 6x - 95 = 0$

(b) 5.55

4 (a) $2x(x - 2) = 51$

$2x^2 - 4x - 51 = 0$

(b) 6.15

(c) It has to be greater than 2 and cannot be negative

33. Completing the square

1 (a) $(x - 5)^2 - 24$ (b) $(x - 1)^2 + 6$ (c) $(x + 3)^2 - 13$

2 (a) $2(x + 3)^2 - 9$ (b) $3(x + 1)^2 - 5$ (c) $4(x + 1)^2 - 15$

3 (a) 1.16 and −5.16 (b) 6.54 and 0.459 (c) 0.25 and −2

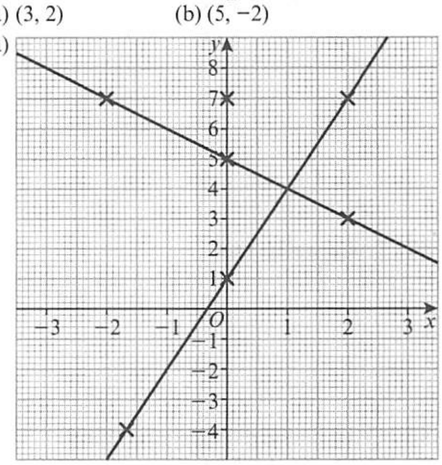
4 (a) $-2 \pm \sqrt{7}$ (b) $\frac{-5 \pm \sqrt{73}}{4}$

5 (a) $3\left(x + \frac{5}{6}\right)^2 - \frac{13}{12}$ (b) $\frac{-5 \pm \sqrt{13}}{6}$

34. Simultaneous equations 1

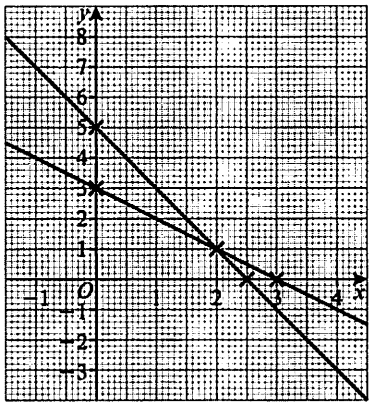
1 (a) (3, 2) (b) (5, −2)

2 (a)



(1, 4)

(b)



(2, 1)

3 (a) $6r + 2f = 72$

Dividing by 2,

$3r + f = 36$

(b) $r = 7$p and $f = 15$p

35. Simultaneous equations 2

1 $x = 1$ and -2

$y = 2$ and -1

2 (0.5, 5) and (3, 0)

3 (1, 3) and (2, 2)

4 (−3, −6) and (6, 3)

36. Equation of a circle

1 (a) $x^2 + y^2 = 36$

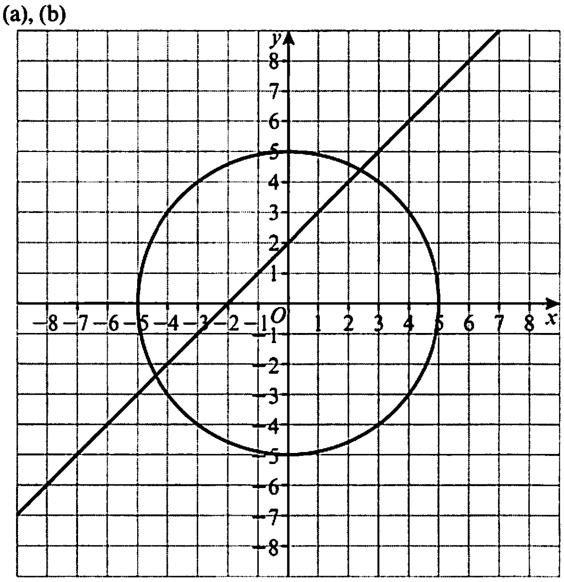
(b) $x^2 + y^2 = 169$

(c) $16x^2 + 16y^2 = 25$

2 (a) (1, −1)

(b) The line is a tangent to the circle

3 (a), (b)



(c) (2.4, 4.4) and (−4.4, −2.4)

4 (a) $(4)^2 + (-6)^2 = 52$ (b) $y = \frac{2}{3}x - \frac{26}{3}$

37. Inequalities

1 (a) $x \leqslant 4$ (b) $x > 3$ (c) $x > -11$

2 (a) 9 (b) 4

3 2

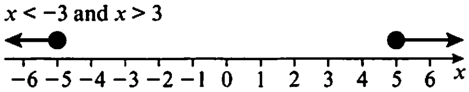
4 (a) $x \geqslant 4$ (b) $x < -1.2$

5 (a) $4x - 14 < 53$ (b) $x < \frac{67}{4}$ so $x = 16$

38. Quadratic inequalities

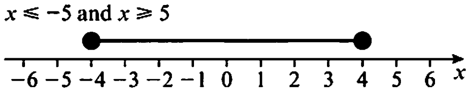
1 (a) $x < -3$ and $x > 3$

(b)



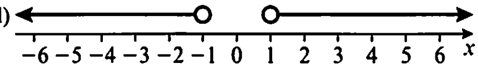
$x \leqslant -5$ and $x \geqslant 5$

(c)



$x \geqslant -4$ and $x \leqslant 4$

(d)



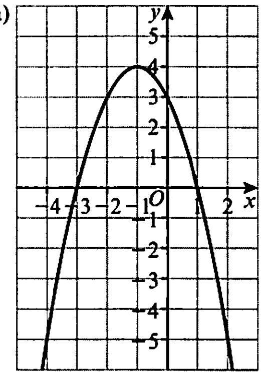
$x < -1$ and $x > 1$

2 (a) $x < 3\sqrt{3}$ and $x > -3\sqrt{3}$ (b) $x \leqslant -4\sqrt{3}$ and $x \geqslant 4\sqrt{3}$

3 (a) $x > -1$ and $x < 5$ (b) $x \leqslant 3$ and $x \geqslant 8$

(c) $x \geqslant -2$ and $x \leqslant 5$ (d) $x < \frac{1}{2}$ and $x > \frac{3}{2}$

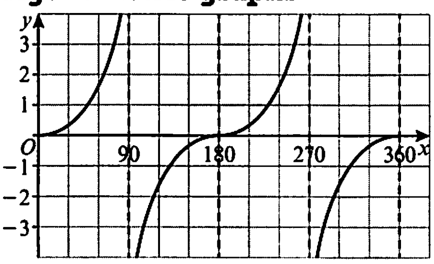
4 (a)



(b) $x > 1$ and $x < -3$

39. Trigonometric graphs

1 (a)



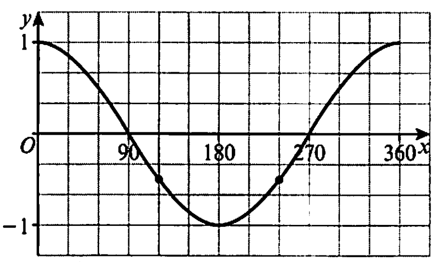
(b) $x = 90°$ or $x = 270°$

2 (a) 30° and 150°

(b) 30° and 330°

(c) 34.2° and 214.2°

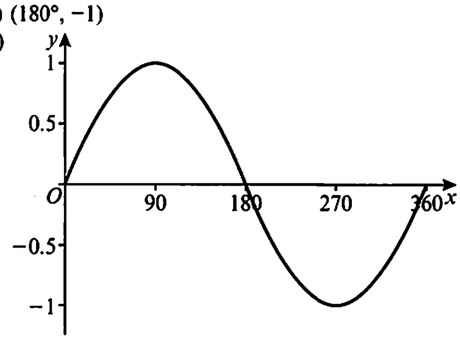
3 (a)



(b) 120° and 240°

(c) (180°, −1)

4 (a)

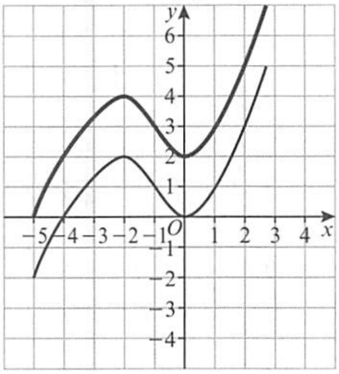


(b) 60° and 120°

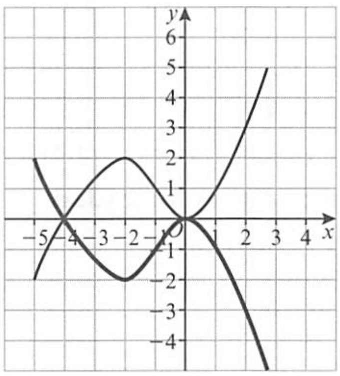
(c) (90°, 1) and (270°, −1)

40. Transforming graphs

1 (a)



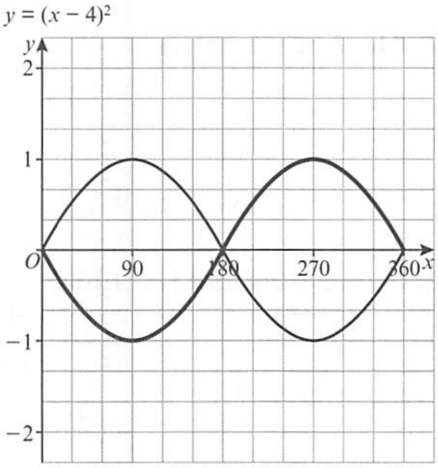
(b)



2 (a) (6, 3)
(b) (2, 7)
(c) (2, −3)
(d) (−2, 3)

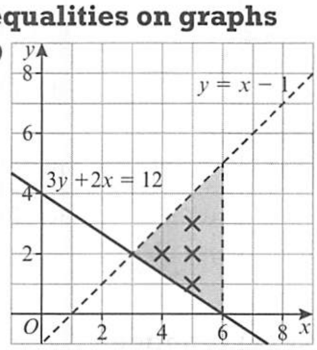
3 $y = (x - 4)^2$

4

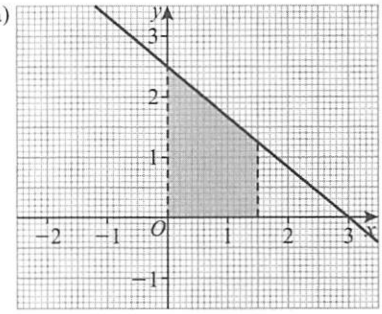


41. Inequalities on graphs

1 (a), (b)

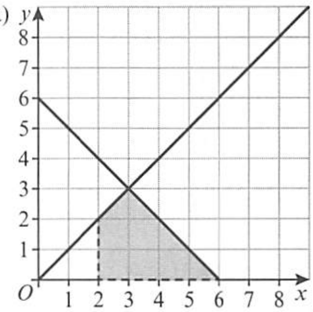


2 (a)



(b) (1, 1)

3 (a)

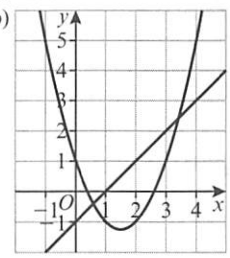


(b) (3, 1) (3, 2) (3, 3) (4, 1) (4, 2) (5, 1)

42. Using quadratic graphs

1 (a) 2.6 and 0.4

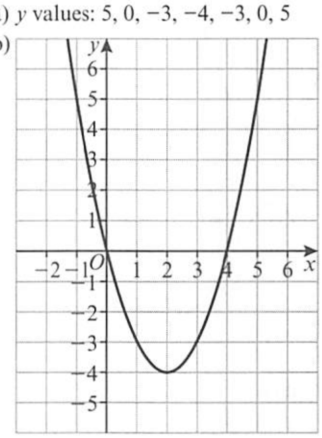
(b)



0.6, 3.4

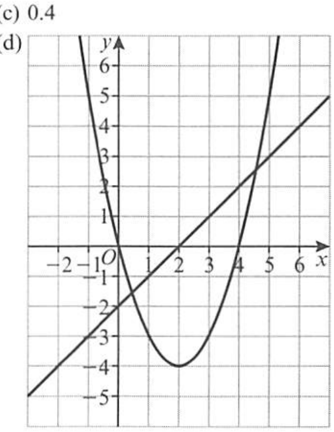
2 (a) y values: 5, 0, −3, −4, −3, 0, 5

(b)



(c) 0.4

(d)



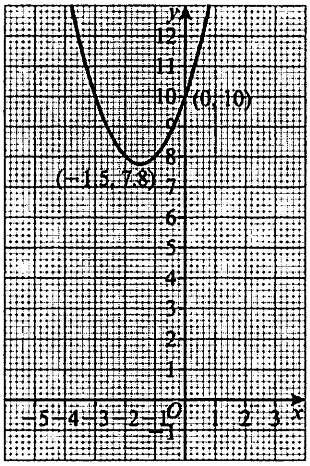
0.4,4.6

43. Turning points

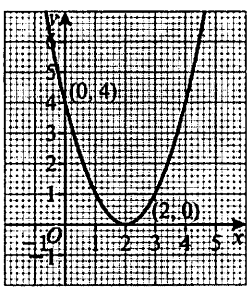
1 (a) (−2, −10) (b) (−1, −5)
(c) $\left(-\frac{3}{2}, -\frac{1}{4}\right)$ (d) $\left(-\frac{3}{4}, -\frac{65}{8}\right)$

2 (a) $p = -2$ and $q = -2$ (b) −2

3 (a)



(b)



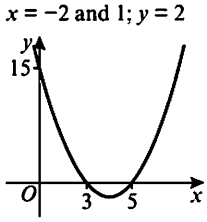
4 (a) (1, 2) (b) $y = (x - 1)^2 + 2$

44. Sketching graphs

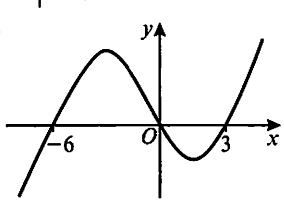
1 (a) $x = -7$ and 3; $y = -21$ (b) $x = -3$ and $\frac{1}{2}$; $y = 3$

2 (a) $x = -3$, 0 and 3; $y = 0$ (b) $x = -5$, 2 and 4; $y = 40$
(c) $x = -2$ and 1; $y = 2$ (d) $x = -4$ and 1; $y = -16$

3 (a)



(b)



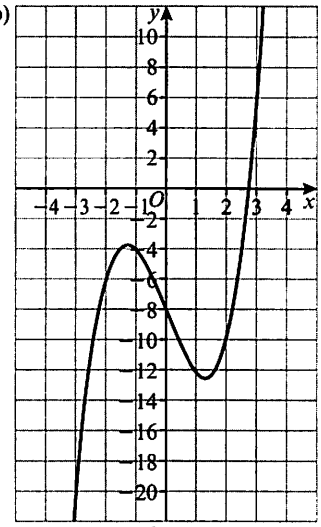
45. Iteration

1 (a) $x^3 = 3 - x$ so $\sqrt[3]{3 - x}$ (b) 1.21

2 (a) $x^3 = 7x - 2$ so $\sqrt[3]{7x - 2}$ (b) −2.778

3 (a) y values: −20, −6, −4, −8, −12, −10, 4

(b)



(c) $x^3 = 5x + 8$ so $\sqrt[3]{5x + 8}$ (d) 2.803

46. Rearranging formulae

1 $x = 3(y - 4)$

2 $h = \dfrac{2d^2}{3}$

3 $w = \dfrac{120P - 10}{3}$

4 $x = \dfrac{P - \pi r - 2r}{2}$

5 $x = \dfrac{Mt - t^2}{1 - M}$

6 $t = \dfrac{yp}{2p + y}$

47. Algebraic fractions

1 (a) $\dfrac{7}{8x}$ (b) $\dfrac{x + 7}{6}$ (c) $\dfrac{x - 7}{(x + 3)(x - 2)}$ (d) $\dfrac{x^2 + y^2}{xy}$

2 (a) $\dfrac{x + 1}{4}$ (b) $\dfrac{x + 5}{x + 3}$

3 (a) $\frac{3}{5}$ (b) $\dfrac{8}{5r}$ (c) $\dfrac{5}{x^2}$ (d) $\dfrac{x - 4}{3}$

4 (a) $x = \frac{8}{3}$ (b) $x = \frac{1}{16}$ (c) $x = \frac{18}{13}$ (d) $x = 6$

48. Quadratics and fractions

1 (a) $x = 2$ and $x = 6$ (b) $x = 3.5$ and $x = 5$
(c) $x = 0$ and $x = -2$ (d) $x = \frac{3}{8}$ and $x = 1$
(e) $x = \frac{1}{3}$ and $x = \frac{1}{2}$ (f) $x = 13$ and $x = -5$
(g) $x = -1$ and $x = 4$

2 (a) $\dfrac{400}{x}$ (b) $\dfrac{400}{x + 10}$
(c) $\dfrac{400}{x} - \dfrac{400}{x + 10} = \dfrac{40}{60}$ (d) 72.6 km/h

49. Surds 2

4 $a = 7$ and $b = 8$

5 (a) $\frac{1}{2}$ (b) $3\sqrt{2}$

50. Functions

1 (a) (i) 13 (ii) $\frac{1}{3}$ (b) $\dfrac{2}{2x - 1}$

2 (a) −11 (b) 81 (c) 126 (d) $4x^3 + 1$

3 (a) 3 (b) $\dfrac{x}{4 - x}$

4 $x = 1$ and $x = \frac{1}{2}$

51. Inverse functions

1 (a) $\dfrac{x + 5}{3}$ (b) $\dfrac{3(x - 7)}{4}$ (c) $\dfrac{7}{1 - x}$

2 (a) $\dfrac{x - 4}{3}$ (b) $\dfrac{x + 1}{4}$ (c) $\dfrac{7x - 13}{12}$

3 (a) $\dfrac{x - 1}{2}$ (b) 5

4 (a) $\frac{5}{3}$ (b) $\dfrac{x}{x - 1}$

52. Algebraic proof

1 LHS $= 4n^2 - 2n - 2n + 1 + 4n^2 + 2n + 2n + 1$
$= 8n^2 + 2 = 2(4n^2 + 1) =$ RHS

2 $5x - 5c = 4x - 5$
$x = 5c - 5$
$x = 5(c - 1)$

3 $(3x + 1)^2 - (3x - 1)^2$
$= (3x + 1)(3x + 1) - (3x - 1)(3x - 1)$
$= 9x^2 + 3x + 3x + 1 - (9x^2 - 3x - 3x + 1)$
$= 9x^2 + 3x + 3x + 1 - 9x^2 + 3x + 3x - 1$
$= 12x = 4(3x)$

4 $2n + 2n + 2 + 2n + 4 = 6n + 6 = 6(n + 1)$

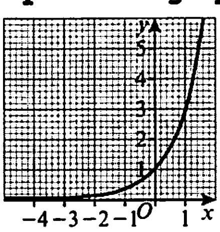
5 $n(n + 1) + (n + 1)(n + 2) = n^2 + n + n^2 + 2n + n + 2$
$= 2n^2 + 4n + 2 = 2(n^2 + 2n + 1)$
$= 2(n + 1)^2$

6 $\frac{1}{n} - \frac{1}{n+1} = \frac{n+1-n}{n(n+1)} = \frac{1}{n(n+1)}$

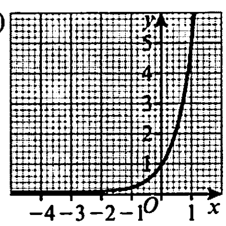
7 $(10a + b)^2 - (10b + a)^2$
$= 100a^2 + 20ab + b^2 - 100b^2 - 20ab - a^2$
$= 99a^2 - 99b^2 \quad = 99(a^2 - b^2)$

53. Exponential graphs

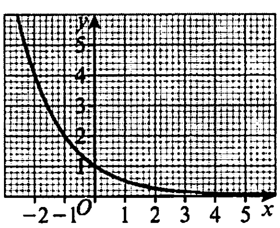
1 (a)



(b)



(c)



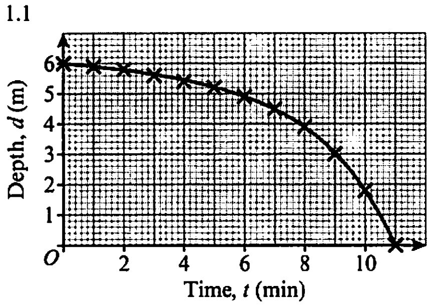
2 $a = 4$ and $k = 1$

3 (a) $2000 \times 1.2^3 = 3456$
(b) $a = 2000$ and $b = 1.2$
(c) $k = 1.728$

54. Gradients of curves

1 (a) 1.1

2 (a)



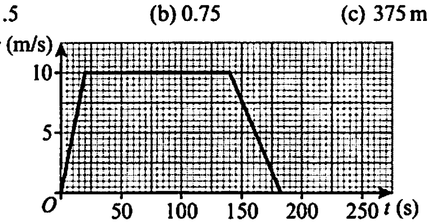
(b) 0.55 (c) 0.2

3 (a) 0.52 (b) 0.58

55. Velocity–time graphs

1 (a) 1.5 (b) 0.75 (c) 375 m

2 (a)



(b) 1500 m

3 (a) 3.2
(b) It is the acceleration of the object at $t = 3$ seconds

56. Areas under curves

1 (a) 19.5
(b) Underestimate, because the trapeziums are below the curve

2 (a) 6.75
(b) Underestimate, because the trapeziums are below the curve

3 28.5

57. Problem-solving practice 1

1 $x = 4$

2 $x = 5$ and $y = 14$

3 D is at (3, 6)
$m_{AB} = \frac{3}{2}$, $m_{CD} = -\frac{2}{3}$, $m_{AB} \times m_{CD} = -1$
So, no she is incorrect, CD is perpendicular to AB

4 $n^2 - 3n + 3$

58. Problem-solving practice 2

5 1 m

6 Area under graph is 47 m
No, the driver does not stop before the traffic lights

7 (4,7) and (−1, −8)

RATIO & PROPORTION

59. Calculator skills 2

1 £633.60

2 (a) 31 200 000 (b) 13.9%

3 (a) 12.9% (b) 19 000 000

4 (a) 675 euros (b) £96
(c) Yes, because the calculations can be done without a calculator and 1.5 euros to the pound is a good approximation.

60. Ratio

1 (a) £20 : £30 (b) £100 : £250 : £400

2 (a) 5 parts = £120
1 part = £24
Amish = 3 × £24 = £72
(b) (3 × 24) + (4 × 24) + (9 × 24) = £384

3 (a) 28 g (b) 42 g

4 £4400

61. Proportion

1 20 days

2 Large basket

3 8

4 No, because she has only 200 ml of milk and needs 250 ml

5 Germany: 4.95 lb = £20.37 so 1 lb = £4.11
Scotland: 2.75 lb = £12.42 so 1 lb = £4.52
Cheaper in Germany

62. Percentage change

1 Mega-jet is cheaper

2 (a) 34.8% (b) 28%

3 £23 320

4 £1454.64

5 33.3%

63. Reverse percentages

1 £60

2 £33 250

3 £33 600

4 750

5 No, she earned £518.52 last year

6 Alison invested £1650 and Nav invested £1680, so Nav invested more

64. Growth and decay

1 £17 569.20

2 3

3 (a) £12 528.15 (b) £6332.78

4 (a) 6.5% (b) £2815.71

5 It is worth £110.74